Bucklin and Condorcet Consistency of Dynamic Voting

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ABSTRACT

Background: When presented with many alternatives, a voter may face considerable difficulty, cognitive or computational, in expressing their full preferences over all candidates. This highlights the importance of exploring voting mechanisms that enable voters to assess and convey their preferences in real time. In this paper, we introduce a family of natural dynamic processes where voters do not need to submit their full preference order at the outset. Instead, they gradually disclose their preferences by approving additional candidates (in single or multiple steps) as they are prompted over time (according to some random or predetermined order). The process concludes, and the winner is declared, when a candidate surpasses the required vote quota.

Objectives and Research Questions: We assess the quality of outcomes for such procedures. Specifically, we measure how "efficient" they are in terms of approximating Bucklin and Condorcet ideals (via Bucklin distance and Condorcet distance).

Methods: The paper combines formal theoretical analysis with empirical evaluation on generated preference profiles.

Results: We show that deciding whether a given candidate can be a winner of a multistep dynamic process is NP-hard for any quota < 1, while single-step dynamics output a winning set coinciding with that of an extended version of the Bucklin rule where any candidate that meets the quota is tied for winning. As a conceptual contribution, we introduce a new measure of voting procedure efficiency, called Bucklin distance, and compare it with a similar metric, Condorcet distance, inspired by but differing from the previously explored notion of Condorcet efficiency. Since single-step dynamics show zero Bucklin distance and the classical Bucklin rule is not Condorcet-consistent, we demonstrate that the extended version of Bucklin, as well as our dynamic voting procedures, may also break the Condorcet principle. However, empirically, we show that in about 70% of instances this is not the case, and in about another 25% it is enough to exclude either one candidate or one voter from consideration, to restore the Condorcet-consistency.

Conclusions: Overall, these results paint a nuanced picture of dynamic approval voting as a practically promising yet theoretically delicate approach. They reveal that seemingly simple multistep procedures hide substantial computational hardness, while their single-step counterparts align cleanly with an extended Bucklin rule and exhibit strong empirical Condorcet performance despite formal impossibility.

KEYWORDS

Dynamic voting, Bucklin consistency, Condorcet consistency

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1 INTRODUCTION

The study of voting mechanisms has long been a cornerstone of social choice theory, exploring ways to aggregate individual preferences into collective decisions. In traditional voting contexts, voters are often required to submit their full preference rankings over a set of alternatives. However, the cognitive and computational challenges associated with ranking a large number of candidates have motivated research into alternative frameworks that relax this requirement. This paper builds upon and contributes to the growing body of work on dynamic and partial preference elicitation, dynamic voting mechanisms, and efficiency metrics.

In many real-world scenarios, voters are often required to select from a large pool of alternatives, presenting formidable cognitive and logistical challenges. In political elections, particularly in primary races, voters may face a crowded field of candidates, requiring them to evaluate and rank multiple individuals vying for the same position. Similarly, participatory budgeting processes involve community members deciding how to allocate resources among numerous proposed projects, such as infrastructure improvements or social programs. Outside of elections, analogous challenges occur in contexts such as university admissions and job recruitment, where committees must shortlist candidates from an extensive pool of applicants, often based on diverse criteria. Online platforms ask the users to rank or vote on content in real time, e.g., by upvoting posts on Reddit or selecting responses on Stack Overflow. These contexts emphasize the need for voting procedures that accommodate large candidate sets efficiently, enabling voters to express their preferences in a manner that minimizes cognitive burden while ensuring fair and meaningful outcomes.

Indeed, while the assumption that all voters hold and report preferences that are complete rankings over the full set of alternatives

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aligns with classical models of voting in social choice theory [29] and remains valuable for analyzing certain types of applications, it may be violated in real-world settings where agents often possess limited or incomplete preferences. In practical scenarios, an agent may be unable to compare some alternatives [22] or may have to use a particular language to express their preferences (e.g., CP-nets [1, 15, 26, 27]), leading to inherent incompleteness in their preference profile. This could be due to insufficient information, limited computational resources to process available information, or a lack of interest in certain subsets of the candidate space. Such partial preferences are frequently observed in real-world data, as evidenced by many entries in the Preflib repository [16]. To this end, Xia and Conitzer [25] study the necessary and possible winner problems [14] under partial preferences, while Terzopoulou et al. [24] accommodate partial preferences in the context of iterative voting [18, 19].

While these ideas stem from the practical difficulties of requiring the agents to submit complete orders, they are also related to the broader area of preference elicitation. Preference elicitation involves obtaining the agents' preferences incrementally through simple queries—such as "What is the next best option?"—rather than requesting a full ranking all at once, continuing until sufficient information is gathered to identify the winner. Preference elicitation has proven particularly valuable in multiagent systems, with notable applications in combinatorial auctions [21, 23] and voting contexts [3–7, 12, 17, 28].

Against this background, this work presents a family of dynamic voting processes where voters are not required to submit their complete preference order upfront. Instead, they gradually reveal their preferences by approving additional candidates—either individually or in groups—when prompted according to a random or given sequence. The process ends, and the winner is declared, when a candidate exceeds the specified vote quota. In contrast with the somewhat similar paradigm of *fallback voting* [2] that combines ordinal and approval preferences, our procedures always output a winner.[1]

We evaluate the quality of the outcomes generated by the proposed dynamic procedures through both analytical and empirical methods, where our primary goal is reaching a broadly agreed decision. To this end, we focus on the Bucklin and Condorcet outcomes, as ones that ensure broader support by aggregating voter preferences across multiple levels, allowing candidates with widespread appeal to be favored over those with limited but concentrating support. Our analysis shows that for multistep dynamics where voters may approve several additional candidates in one round, determining whether a given candidate is in the set of winners is NP-hard for any quota less than 1. In contrast, single-step dynamics, with one additional approval given in each round, produce a winning set that aligns with an extended version of the classical Bucklin rule, where all candidates meeting the quota are considered tied winners.

[1]The difference with the fallback rule is in the sequential vs. simultaneous fashion in which the voters reveal the next level of preferences. Also, in fallback voting, unlike in our method, the voters have a mix of ordinal and approval preferences, and there may be an insufficient number of approvals to reach the voting quota, so the winner may not necessarily exist.

The Bucklin voting rule offers several advantages, making it a compelling choice in elections. First, it ensures broader support by aggregating voter preferences across multiple levels, allowing candidates with widespread appeal to be favored over those with limited but concentrated support. This reduces the likelihood of polarizing winners, as Bucklin tends to select compromise candidates who are acceptable to a majority (or other desirable threshold) of voters. Additionally, the system mitigates the problem of vote splitting, which can occur in plurality voting, by considering secondary preferences, ensuring that similar candidates do not undermine each other's chances.

This brings the introduction of a novel measure of efficiency for voting procedures, *Bucklin distance*, which is our conceptual contribution. Taking inspiration from the literature on control in elections [9], Bucklin distance is defined as the smallest number of voters or candidates that need to be excluded from a voting profile, in order to obtain the Bucklin set of winners. We then compare the Bucklin distance with an analogously defined measure based on the Condorcet principle, *Condorcet distance*, in spirit with the earlier explored *Condorcet efficiency* [13, 20] that, in contrast, quantifies the alignment of the voting rule with the principle by measuring how often it selects the Condorcet winner (if one exists) in a given set of election scenarios.

Although single-step dynamics always achieve zero Bucklin distance and multi-step dynamics do so in about 96% of instances, none of our procedures show Condorcet distance of 0. However, empirical results indicate that despite negative theoretical findings, in practice, our dynamic processes are *almost* Condorcet-consistent. Specifically, in approximately 70% of cases, the Condorcet principle is not violated; in the remaining profiles, the Condorcet distance is often 1, which means that restoring Condorcet consistency requires removing just a single candidate or voter from consideration. Larger distances, as expected, appear in profiles where Condorcet winner did not initially exist. Importantly, the Condorcet distance is defined for these profiles and thus allows for their evaluation, while the earlier considered measure of Condorcet efficiency does not apply to them. The distinctive feature of our procedures is that they will return a very close Condorcet winner even when a Condorcet winner does not exist in the original profile.

2 MODEL AND PRELIMINARIES

In this section, we formally define the model and concepts relevant to our study. We first introduce the setting for Dynamic Voting (DV) and several variants for its implementation. We then define the Bucklin and Condorcet distances.

2.1 Dynamic voting

An *election* is a pair $E = (C, V)$ with a finite set of alternatives (or *candidates*) C and a finite set of *voters* V such that $C \cap V = \emptyset$. Let $|C|$ denote the number of candidates and $|V|$ the number of voters. A *voting rule* $\mathcal{R}$ is a function that determines the winner(s) of any given election: $\mathcal{R}(E) \subseteq C$.

We consider dynamic voting systems in the form of iterative processes in which voters are asked to expand their ballots (sets of approved candidates) in turns, until some candidate is approved by a required quota q of voters; this candidate is declared the winner.

Given the set $\mathcal{L}(C)$ of all linear orders over C, each voter $v \in V$ is associated with a *preference order* $\succ_v \in \mathcal{L}(C)$, which they gradually assess and communicate to the mechanism by approving additional candidates from this order as the election progresses. The list $P = (\succ_v)_{v \in V}$ is referred to as the voters' *preference profile*. Importantly, although we assume ordinal voter preferences, and these preferences affect the progression of the voting dynamics, the aggregation of preferences is based solely on the number of approvals received by each candidate.

Let $p_t(v)$ be the variable that denotes the furthest position in voter v's preference order $\succ_v$ that has been approved by the end of round t. The *dynamic voting* setting is defined as follows:

(1) To begin with (in round $t = 0$), each voter $v \in V$ approves their top candidate: $p_0(v) = 1$ for all $v \in V$.
(2) If there is a candidate whose total number of approvals meets the required quota $\frac{|V|}{2} < q \leq 1$, the process ends and the winner is announced; otherwise, the process proceeds to the next round.
(3) In each round $t \geq 1$ of the process, the procedure selects an *active voter*, v_t, to expand their ballot.
(4) The active voter v_t approves the set of one or more additional positions in their preference order: $p_t(v_t) = p_{t-1}(v_t) + x_t$ where $x_t \geq 1$ is determined in a manner specified below; for other voters $v \neq v_t$, we have $p_t(v) = p_{t-1}(v)$.
(5) Return to step (2).

To complete the definition, we need to specify the two components of the process used in steps (3) and (4), respectively: the method by which the active voter is selected and the way in which the active voter expands their ballot. Let $\xi : \mathbb{N} \to V$ denote the *activation function* that selects the active voter in round $t \geq 1$ of the process; we shall term the resulting sequence $s = (\xi(t))_t$ of active voters the *activation sequence*. We consider the following active voter selection schemes:

(P) Predetermined: Each voter is activated in turn according to a given fixed ordering, $\rhd$, skipping over voters who have approved all candidates.
(R) Random: Randomly selects one of the voters among those with fewest approved candidates: $v_t \in \arg\min_{v \in V} p_{t-1}(v)$.
(H) Hybrid: Activates voters with fewest approved candidates according to a given fixed ordering.

An active voter in round $t \geq 1$ will expand their ballot in one of the following manners:

(+) Single-step: The active voter approves one additional candidate: $x_t = 1$.
(++) Multistep: The active voter approves one or more additional candidates, as necessary to approve their favorite candidate among those who so far received approval from other voters but not from v_t[2]: $x_t \geq 1$. Let $p_v(c)$ denote the position of candidate $c \in C$ in the preference order $\succ_v$ of voter $v \in V$, and let $C_j(v) = \{c \in C | p_v(c) \leq p_j(v)\}$ be the set of candidates approved by voter $v \in V$ up to round $j \leq t$. Let $C_j = \cup_{v \in V} C_j(v)$ denote the set of candidates that have received at least one approval from the entire set of voters V up to the round j. In round t, let c_t be the favorite candidate of the active voter v_t in $C_{t-1} \setminus C_{t-1}(v_t)$: $c_t = \arg\min_{c \in C_{t-1} \setminus C_{t-1}(v_t)} p_{v_t}(c)$. Then $p_t(v_t) = p_{v_t}(c_t)$ (or $x_t = p_{v_t}(c_t) - p_{t-1}(v_t)$)—that is, voter v_t will approve c_t and, if there are any, all candidates $c \succ_{v_t} c_t$ not yet approved by anyone.

In what follows, we use abbreviation DV for dynamic voting and, respectively, DP+, DP++, DR+, DR++, DH+, DH++ for each type of dynamic processes as defined above. Note that each such process ends within $|C| \cdot |V|$ (or fewer) rounds.

2.2 Bucklin and Condorcet distances

The two paradigms in the focus of our study are Condorcet principle and Bucklin voting system. Specifically, we shall be interested in establishing the distance between the outcomes of our procedure and Bucklin and Condorcet winners.

Bucklin rule. Given election $E = (C, V)$ and an integer $1 \leq k \leq |C|$, a candidate c is called a *k-majority winner* if more than $\frac{|V|}{2}$ voters rank c among their top k choices. Let k' be the smallest integer for which there exists at least one k'-majority winner. The *Bucklin score* of a candidate c is the number of voters who place c in top k' positions. The *Bucklin winners* are the candidates with the highest Bucklin score. The *simplified* Bucklin winners are all k'-majority winners [8]. In this work, we shall consider this simplified version of Bucklin; moreover, we *extend* the rule to replace the majority threshold with whichever desirable *quota* $\frac{|V|}{2} < q \leq 1$ of voters—thus, any candidate that meets the quota is considered the winner for E.

Condorcet principle. Given election $E = (C, V)$, a candidate $c \in C$ is a *Condorcet winner* if c is preferred to every other candidate in pairwise comparisons by a majority of voters. A voting rule satisfies the *Condorcet principle* if it guarantees that, whenever a Condorcet winner exists, that candidate is chosen as the election winner.

Bucklin/Condorcet distance. To quantify how consistent dynamic voting is with the Condorcet principle or the Bucklin rule, for a given election $E = (C, V)$ with the preference profile P, we calculate the smallest number $k \geq 0$ of agents (candidates or voters) that need to be removed from the set C or V, respectively, in order to create a reduced profile P' such that the winners of dynamic voting on P' are aligned with the Bucklin rule or the Condorcet principle. We call k the *candidate* (resp., *voter*) *Bucklin* (resp., *Condorcet*) *distance* for instance E. The rule is called *Bucklin* or *Condorcet consistent* if its respective distance is zero for all instances.

We consider three types of distances, indicating scenarios where: 1) the dynamic voting winners in the original profile are the Bucklin (respectively, Condorcet) winners in the ablated profile; 2) the outcomes by Bucklin or Condorcet in the original profile are the dynamic winners in the ablated profile; and 3) the sets of dynamic and Bucklin (respectively, Condorcet) winners in the ablated profile coincide:[3]

(1) $DV(P) = Bucklin(P')$ (resp., $DV(P) = Cond(P')$)
(2) $DV(P') = Bucklin(P)$ (resp., $DV(P') = Cond(P)$)

[2]Consider the following example. Suppose you and your colleagues are choosing a time slot for a meeting in a Doodle poll. Assume someone has voted for 9am, which you accept but find 10am, 11am, and 12pm more preferable. With such preferences, you will approve all these 4 options in the poll.

[3]Note that for any of these types, it induces distance zero if and only if the all three types do so.

(3) $DV(P') = Bucklin(P')$ (resp., $DV(P') = Cond(P')$)

where $DV(\cdot)$, $Bucklin(\cdot)$ and $Cond(\cdot)$ are the sets of dynamic, Bucklin and Condorcet winners. By dynamic winners we mean the winners for all activation sequences of a given process, that is: $DV(P) = \{DV(P,s)\}_{\forall s}$ where $DV(P,s)$ is the profile P winner for activation sequence $s \in S(DV)$ from the set of all activation sequences, $S(DV)$, for dynamic DV. As we show next, each such sequence will contribute exactly one winner to the set.

3 THEORETICAL ANALYSIS

We prove theoretical results for predetermined and random activation functions and add the hybrid scheme to the consideration of the empirical analysis presented in the following section. We start with Lemma 1 below showing that our dynamic processes always have a winner, which is unique for any given activation sequence. We then use this lemma to prove Theorem 1 about the Bucklin consistency of single-step dynamics. In contrast, multistep dynamics may not necessarily produce a Bucklin winner (Proposition 1), while determining whether a given candidate can win the election following such a multistep procedure generally appears to be NP-hard (Theorem 2). Finally, none of the processes presented are Condorcet-consistent (Proposition 2), which opens the room for empirical analysis in the next section.

Lemma 1. *For each of the dynamics DP +, DP++, DR +, DR++, any activation sequence s induces a voting process that ends with a unique winner, w, who is the sole candidate approved by the active voter in the last round of prompting.*

Proof. Note that if the winner is determined after round $t = 0$ then it is unique due to $\frac{|V|}{2} < q$. Otherwise, for single-step dynamics, assume on the contrary there are more than one dynamic winners, w_1 and w_2, and assume that the last approved candidate wasn't w_2. Then, w_2 reached the quota of votes before the last step of dynamics, a contradiction. Hence, w_1 is the only winner. Again, if it is not the last approved candidate then it reached the vote quota before the last step, a contradiction. For multistep dynamics, note that in each round, only one candidate with non-zero score gets approved, hence only this candidate can be the winner. □

3.1 Bucklin (in)consistency

Single-step dynamics.

Theorem 1. *Single-step dynamics are Bucklin-consistent.*[4]

Proof. To show this, we prove that for any election $E = (C, V)$ with preference profile P the following holds:

(1) For all activation sequences s, the dynamic winner is a Bucklin winner: $\forall s \in S(DV), DV(P,s) \subseteq B(P)$.
(2) For any Bucklin winner $w \in B(P)$ there exists an activation sequence $s \in S(D)$, such that w is the dynamic winner for s: $\forall w \in B(P), \exists s \in S(DV), w = DV(P,s)$.

We need the following lemma:

Lemma 2. *For single-step dynamics DP + and DR +, and any two voters $v_1, v_2 \in V$, we have $|p_t(v_1) - p_t(v_2)| \leq 1$ in any round t of the process.*

Proof. Assume on the contrary, that there exist v_1, v_2, t with $p_t(v_1) - p_t(v_2) \geq 2$.

For DR+, let $l \leq t$ be the last round where voter v_1 approved a candidate. Since $l \leq t$, we have $p_l(v_2) \leq p_t(v_2)$, and so $p_l(v_1) - p_l(v_2) \geq 2$. It then follows that $p_{l-1}(v_1) - p_{l-1}(v_2) \geq 1$, and therefore the voter v_1 could not be the one prompted in round l, a contradiction.

In DP+, prompting follows a predetermined order of voters. After each full cycle in this order (that is, after each series of $|V|$ rounds), the number of candidates approved by each of the voters increases by 1. We thus have $p_t(v) = \lfloor \frac{t}{|V|} \rfloor$ or $p_t(v) = \lceil \frac{t}{|V|} \rceil$ for all $v \in V$, implying $|p_t(v_1) - p_t(v_2)| \leq 1$ for any $v_1, v_2 \in V$. □

We now proceed to prove the theorem.

(1) By Lemma 2, at any time t, the number of candidates approved so far by any pair of voters may differ by at most 1. Thus, at the end of the process, there maybe only two types of voters: with A approvals or with $A + 1$ approvals.

By Lemma 1, let $v \in V$ be the voter who approves the unique dynamic winner $w \in C$ in the last round, T.

Let $p_T(v) = A + 1$. In the dynamic process, no other candidate's score has reached the quota. Therefore, when all the voters approve A candidates, no one reaches the quota, and when all the voters approve $A + 1$ candidates then at least the dynamic winner reaches the quota. That means, that the Bucklin winner will be determined by the $(A + 1)$-approval score and the dynamic winner will be among the Bucklin winners.

Let $p_T(v) = A$. Then, until the last round v had $A - 1$ approvals, and so none of other voters approves $A + 1$ candidates (otherwise, we have a contradiction to Lemma 2). That means, that the Bucklin winner will be determined by the A-approval score and the dynamic winner will be among the Bucklin winners.

(2) Let k be the smallest integer for which the quota is reached in Bucklin, and let $w \in C$ be one of the Bucklin winners. Then, for the DP+ dynamic, the following order of prompting will make w the dynamic winner:

$$\text{voters with } w \text{ in position } k \rhd \text{all other voters}$$

For DR+, after all the voters have $k - 1$ approvals, randomly select voters that have w in the kth position, until it reaches the quota (and becomes a dynamic winner).

□

Multistep dynamics.

Proposition 1. *Multistep dynamics may fail to be Bucklin-consistent.*

Proof. Consider the following example with $|V| = 5$ voters and preferences given as following columns:

a	b	c	f	d
c	d	b	a	x
d	c	a	x	y
x	x	x	y	b
		…		

Here, a, c and d are Bucklin winners, but candidate b will be selected by a multistep dynamics as, after the initial round where each of

[4] As noted earlier, we consider the extended, simplified version of the Bucklin rule.

these candidates get 1 vote, b will be further approved by voters 3 and 5 and reach the quota before the Bucklin winners, regardless the order of prompting. ◻

THEOREM 2. *Given election E=(V,C) with preference profile P, a candidate $w \in C$, and a multistep voting dynamics $DV \in \{DP++, DR++\}$, it is NP-complete to determine whether $w \in DV(P)$.*

PROOF. Given an activation sequence s, it is easy to check whether w is the dynamic winner. Indeed, by Lemma 1, we have $w \in DV(P)$ if it is the last candidate that gets approved in the given sequence, so the problem is in NP.

For hardness, we provide a reduction from a restricted variant of the classic EXACT 3-COVER (X3C) problem. An instance of this problem is given by a set of ground elements $A = \{a_1, \ldots, a_{3r}\}$ and a family $\mathcal{Z} = \{Z_1, \ldots, Z_p\}$ of 3-element subsets of A; let $Z_\ell = \{a_{i_\ell}, a_{j_\ell}, a_{k_\ell}\}$ for some $i_\ell, j_\ell, k_\ell \in \{1, \ldots, 3r\}$. It is a "yes"-instance if there exists a subfamily $\widehat{\mathcal{Z}} \subset \mathcal{Z}$ such that $\cup_{Z \in \widehat{\mathcal{Z}}} Z = A$ and $Z_i \cap Z_j = \emptyset$ for all $Z_i, Z_j \in \widehat{\mathcal{Z}}$ with $i \neq j$; otherwise, it is a "no"-instance. We additionally assume that $r \geq 2$ and that each element of A is contained in exactly three distinct sets in $\mathcal{Z}$; note that this implies that $p = 3r$. This restricted variant of X3C, which we shall refer to as RX3C, has been shown to be NP-complete by [10].

Given an instance $(A, \mathcal{Z})$ of RX3C with $|A| = |\mathcal{Z}| = 3r$, we construct an election $E = (V, C)$ with $|C| = 3r + 9$ candidates $C = A \cup \{b, c\} \cup \{d_l\}_{l=1..7}$ and $|V| = 3r + q$ voters. Note that for $q > \frac{|V|}{2}$ we have $q > 3r$. The preference profile P is specified in Table 1 and is constructed in a way that only candidates b or c can win the election. Our goal will be to check if b is the winner; whenever this is not the case, the winner will be c. We show full proof for DP++, then comment how it adapts to DR++.

In round $t = 0$, all voters approve their top choice candidate. As a result, the candidates with non-zero scores will be b, c and $\{d_l\}_{l=1..7}$, and so we have a candidate with non-zero score in the 2nd position of each vote.

In the first cycle of the activation sequence, the candidates in position 2, and only them, will get approved. The scores after this cycle are:

$score(c) = q - 1$
$score(b) = q - r$
$score(d_1) = r$
$score(d_l) \leq \frac{1}{6}3r = \frac{r}{2}\ \forall l = 2..6$
$score(d_7) \leq \frac{1}{6}3r + 1 = \frac{r}{2} + 1$
$score(a_x) = 0\ \forall x = 1..3r$

In the next cycle of the sequence, the candidates in positions ≥ 3 get approved. If all the voters get prompted, then candidate c will receive additional vote from Block 2 and reach the quota.

In Block 3, we have candidates $\{d_l\}_{l=2..6}$ (with non-zero score) in the 3rd position, and so only them will get votes from this block. Their current score is:

$score(d_l) \leq \frac{q}{5} + \frac{r}{2} < \frac{q}{5} + \frac{q}{6} = \frac{11q}{30}\ \forall l = 2..6$

Candidates $a_x\ \forall x = 1..3r$ can only receive votes from Block 1 in this cycle, and so their current score is:

$score(a_x) \leq 3r < q\ \forall x = 1..3r$

That is, none of $\{d_l\}_{l=2..6}$ or $a_x\ \forall x = 1..3r$ is a winner.

Finally, if candidate b receives r votes in Block 1, it reaches the quota. Therefore, either candidate b or candidate c can win the election.

If the winner is b, this means that it received r votes from Block 1 before c received 1 vote from Block 2. Note that in these r votes all the candidates up to b (including) get approved. We show that these r votes represent a cover. To this end, we need to check that all candidates a_x in these votes are distinct. On the contrary, assume that the same candidate a_x appears in two of these r votes above b. Consider the copy that gets approved first in the current cycle. But then it means that in the other vote containing the same a_x candidate b will not be approved, as a_x will have a non-zero score. Thus, all the votes where b gets approved correspond to distinct elements a_x (of which we have exactly $3r$), and so are a cover.

The opposite direction is straightforward: the sets in the cover should correspond to the voters who get prompted first, and in these votes all the candidates up to b (including) get approved, and so b wins the election.

The proof easily adapts to DR++, as in the second cycle of the activation sequence, in the beginning of which all voters have exactly 2 positions approved, the entire Block 1 will be among the votes that are selected for prompting. ◻

3.2 Condorcet (in)consistency

It is well-known that the classical Bucklin rule fails the Condorcet criterion. Below we formally state this result for extended, simplified Bucklin and extend it to dynamic voting.

PROPOSITION 2. *Extended, simplified Bucklin and dynamic voting procedures (both single- and multistep) are not Condorcet-consistent.*

PROOF. Consider the following example with $|V| \geq 10$ voters and preferences presented in Tables 2 and 3 for quota $q \leq 100\% - 3$ and $q > 100\% - 3$, respectively. Here, candidate b is the unique Bucklin and dynamic winner (for both single-step and multistep dynamics), while the Condorcet winner is c. ◻

To further motivate the empirical analysis in the next section and the earlier presented definitions of Bucklin and Condorcet distances, we show the following Example 1 where the voting dynamics violates the Condorcet principle but the dynamic winner turns into a Condorcet winner if we delete some votes.[5]

EXAMPLE 1. *Consider the following example with $|V| = 7$ voters and preferences given as following columns:*

x	y	a	b	c	d	f
y	x	c	d	a	b	a
c	d	f	f	f	f	b
a	a	d	c	b	a	c
d	c	b	a	d	c	d
b	b	x	x	x	x	x
f	f	y	y	y	y	y

Here, the dynamic winner is f, however it loses 3:4 to candidate a in a pairwise election, and so is not a Condorcet winner. Similarly, f loses to b, c and d. If we delete two votes where f is below these candidates,

[5]Similar examples can be shown to illustrate that deleting voters (or candidates) can make a Condorcet (or Bucklin) winner the dynamic winner in the ablated profile, or even make them coincide.

Table 1: Preference profile

Block 1: $3r$ votes										**Block 2**: 1 vote
d_2	d_3	d_4	d_5	d_6	d_7	d_2	d_3	d_4	$\dots$	d_1
d_3	d_4	d_5	d_6	d_7	d_2	d_3	d_4	$\dots$	$\dots$	d_7
a_{i_1}	a_{i_2}	a_{i_3}			$\dots$				$a_{i_{3r}}$	c
a_{j_1}	a_{j_2}	a_{j_3}			$\dots$				$a_{j_{3r}}$	
a_{k_1}	a_{k_2}	a_{k_3}			$\dots$				$a_{k_{3r}}$	$\vdots$
b	b	b			$\dots$				b	
$\vdots$					$\vdots$				$\vdots$	$\vdots$

Block 3: $(q-1)$ votes											
Block 3.1: $(q-r)$ votes									**Block 3.2**: $(r-1)$ votes		
b	c	c		$\dots$				c	c	$\dots$	c
c	b	b		$\dots$				b	d_1	$\dots$	d_1
d_2	d_3	d_4	d_5	d_6	d_2	d_3	d_4	$\dots$		$\dots$	
$\vdots$						$\vdots$					$\vdots$

Table 2: $q \leq 100\% - 3$

$(50\% - 2)$ votes	3 votes			$(50\% - 3)$ votes	2 votes
c	d_1	d_2	d_3	b	b
b	d_2	d_3	d_1	d_3	d_1
d_1	c	c	c	c	c
d_3	b	b	b	d_1	d_3
d_2	d_3	d_1	d_2	d_2	d_2

Table 3: $q > 100\% - 3$

$(50\% + 1)$ votes	$(50\% - 2)$ votes	1 vote
c	b	d
b	d	b
d	c	c

it will be winning in the respective pairwise elections. We thus can delete votes 1 and 2 to turn candidate f into a Condorcet winner. Note though that in the ablated profile it is no longer the dynamic winner; the new dynamic winner will be a.

It is important for the previous example that deleting votes 1 and 2 is the smallest possible change in the vote, which makes candidate f Condorcet winner of the election. Similarly, a candidate who was Condorcet winner of the full profile could be made a dynamic winner of the election by deleting votes, but may stop being a Condorcet winner if we consider a profile with the smallest possible number of changes.

EXAMPLE 2. *Consider the following example, which organized as in Example 1: the preferences of 7 voters are given as columns.*

c	d	c	b	c	m	f
e	k	d	e	d	n	k
b	m	a	m	a	b	a
k	e	f	f	f	f	b
a	a	b	a	b	a	d
d	b	e	c	e	c	c
f	c	k	d	k	d	e
m	f	m	k	m	e	m
n	n	n	n	n	k	n

In this profile, Condorcet winner is a candidate a. The minimum number of votes needed to be deleted to make a a dynamic winner of the election is 2. While deleting votes 1 and 2 turn candidate a a dynamic winner, it also turns candidate f into a Condorcet winner. That is, it was not only that a is no longer a Condorcet winner, but he was replaced by another candidate.

4 EMPIRICAL ANALYSIS

We assess the performance of Dynamic Voting by measuring its candidate / voter Bucklin / Condorcet distance. To estimate this distance, we draw 1000 preference profiles from impartial culture and exhaustively search through every combination of candidates / voters and verify if deleting these agents will produce winners that are in alignment.

The simulation is implemented in Python 3.10, with the help of the `pref_voting` library [11]. Code will be made available on publication. We fix the simulation at $|V| = 15$ voters and $|C| = 8$ candidates. This is the minimal size at which there is significant difference between the single and multi-step rules. As the simulation iterates through all combinations of agents for deletion, the running time of the simulation rapidly becomes prohibitive at higher voter / candidate counts (see below for details).

For simplicity of exposition, we show results on the third type of distances which is defined as a number of candidates or voters to be deleted so that the dynamic and the Bucklin / Condorcet winner in the ablated profile coincide. The other two types of distances, albeit being qualitatively different, show quantitatively similar results. In all figures, columns appear in the same order (left to right) as keys in the legend (top to bottom).

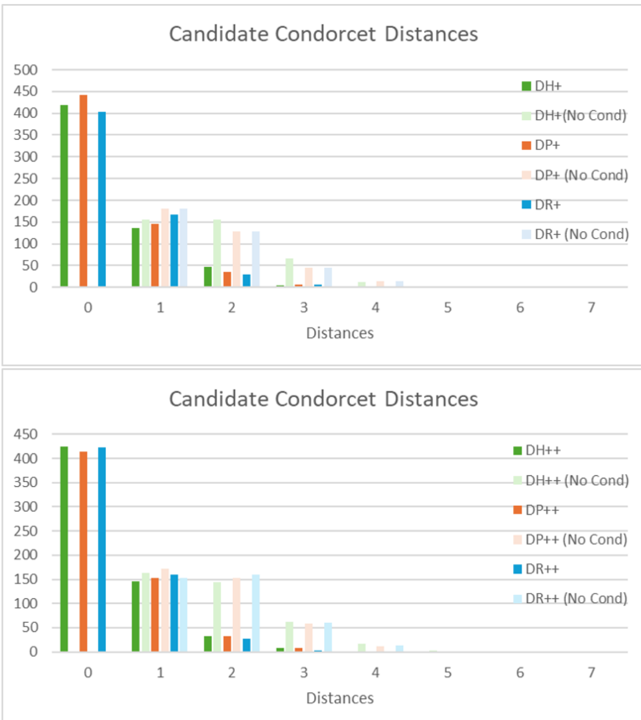

Figure 1: Candidate Condorcet Distance. Top: single-step dynamics; Bottom: multistep dynamics.

Figure 1 shows the histogram for *candidate Condorcet* distances for the six Dynamic Voting procedures. We separate the histograms for instances that did not initially have a Condorcet winner (i.e. $Cond(P)$ does not exist) in a lighter, matching shade. Due to the initial non-existence of a Condorcet winner, it requires several candidate deletions to produce a Condorcet winner at all, hence the higher metric values for these profiles. Once these instances are filtered out, the majority of remaining instances ($66-70\%$) show agreement between Condorcet and Dynamic Voting, and most of the remaining conflicts ($23-27\%$ of filtered profiles) can be resolved with the deletion of 1 candidate.

Figure 2 shows a similar histogram for *voter Condorcet* distances, with the instances with no initial Condorcet winner displayed separately in a lighter, matching shade. We also observe a similar pattern. When instances that have no initial Condorcet winner are set aside, the majority of the remaining instances ($67-71\%$) show agreement between DV and Condorcet, and the majority of the disagreements ($13-23\%$) can be rectified by the deletion of a single agent. It is interesting to note that the distances for profiles with no initial Condorcet distances concentrate around 2, rather than spread more

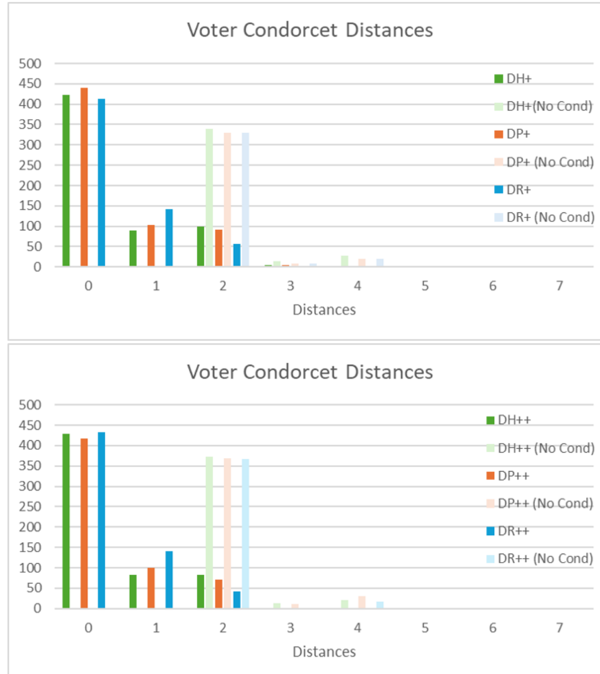

Figure 2: Voter Condorcet Distances. Top: single-step dynamics; Bottom: multistep dynamics.

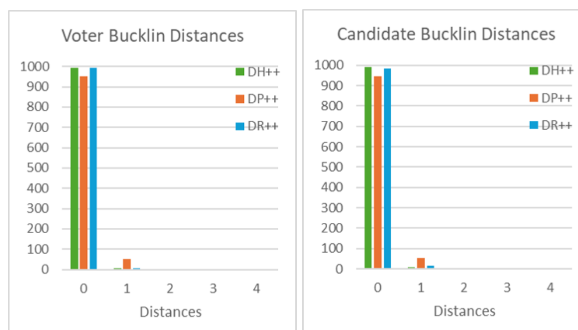

Figure 3: Bucklin Distance. Left: voter distances; Right: candidate distances.

evenly; we surmise that this is due to a tie-breaking effect since the peaks shift to the 1 and 3 positions.

We repeat these experiments to compare Dynamic Voting to the *Bucklin* rule. As single-step procedures are always Bucklin-consistent, we only show results for the multistep dynamics in the histograms of Figure 3. We observe that even the multistep variants are Bucklin-consistent the vast majority of the time (in over 94% of the profiles for DP++, and in over 98% of profiles for DH++ and DR++).

Across these experiments with $|V| = 15$, $|C| = 8$ over 1000 sampled instances, all Dynamic Voting rules arrive at an answer after expanding an average of 38.5 positions. By comparison, expanding all positions would take 120 queries. See Table 4 for details.

DV	DH++	DP++	DR++	DH+	DP+	DR+
Avg	38.28	38.43	38.60	38.51	38.65	38.39
St.dev	7.02	6.55	6.97	6.74	6.95	6.58

Table 4: Number of voter preference positions that need to be expanded for the DV rule to produce a winner ($|V| = 15$, $|C| = 8$; 1000 samples each).

To demonstrate that our results generalize, we focus on the DH++ dynamic voting process and examine the *voter Condorcet* distances for number of voters $n \in \{9, 11, 13, 15, 17, 19\}$. Figure 4 shows that the distribution of distances remains relatively consistent across the range of values for n, with a slight degradation of performance with increasing number of voters. The figure's legend also includes the number of profiles excluded from this plot due to the initial profile not having a Condorcet winner (these range from 359 to 402 out of 1000 profile at each value of n).

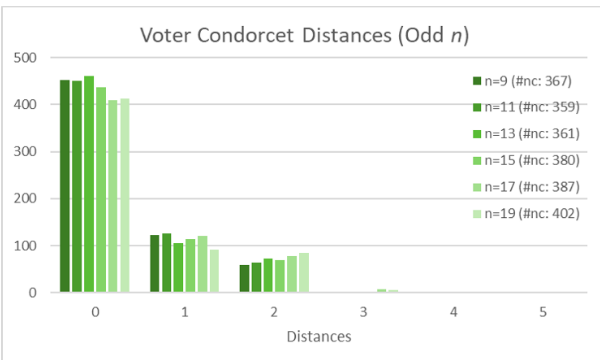

Figure 4: Voter Condorcet distances for odd numbers of voters, using the DH++ process. The legend describes the number of excluded profiles with no initial Condorcet winners ("nc") in each condition.

Figure 5 shows a breakdown of the 1000 sampled profiles, at each voter count. The stripped bars show the number of these profiles that have no initial Condorcet winners (and thus, would have Condorcet distances of greater than 0). The solid bars show the number of profiles where the Condorcet distance is 0. Note the strong parity effect. This is because initial profiles with an even number of voters frequently produce no Condorcet winners. However, if we consider only those sampled profiles with an initial Condorcet winner, and we observe that fraction of those profile that have a Condorcet distance of 0 (depicted as the line in Figure 5 remain consistently high (though we remark that a parity effect still persists).

Finally, Figure 6 shows the run times for the experiments that compare measure the voter Condorcet distances of DH++ across different number of voters n. The linear trend on the logarithmic y-axis show that these experiments have exponential scaling. It is important to note that these run times are not solely due to DH++ (which runs very fast even for large instances) but is dominated by the brute force search on the necessary deletion to catch matching the outcomes of the voting rules.

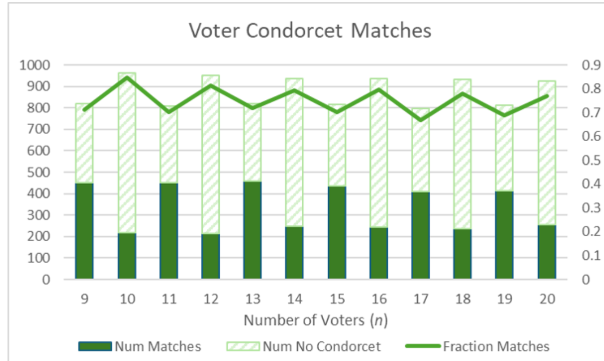

Figure 5: Instances with voter Condorcet distance of 0, both as absolute numbers out of 1000 sampled profiles and as fractions of only profiles with an initial Condorcet winner.

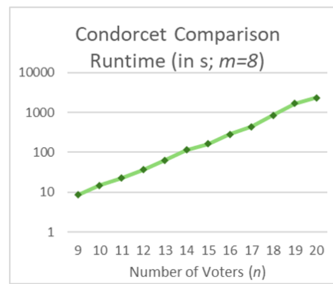

Figure 6: Run times for comparing DH++ with Condorcet. Note the logarithmic y-axis.

5 CONCLUSION

In this work, we introduced a family of dynamic voting processes designed to elicit only the minimal amount of information necessary to determine an election winner. These procedures offer a practical alternative to full preference elicitation by progressively collecting approvals over time. To assess their effectiveness, we proposed two novel evaluation metrics—Bucklin distance and Condorcet distance—that apply to all voting profiles, unlike the more restrictive Condorcet efficiency.

Our empirical findings demonstrate strong alignment between the outcomes of our dynamic procedures and both the Bucklin rule and the Condorcet principle, even though the theoretical complexity of winner determination is high in some cases. Notably, our experiments revealed that, in small-scale elections, the winner could typically be identified after eliciting less than a third of the full preference information. This suggests that dynamic elicitation can yield substantial efficiency gains with small informational burden.

Looking ahead, it would be valuable to apply our proposed distance measures to a broader range of voting rules—both existing and newly developed—to better understand their alignment with normative principles. Furthermore, a formal analysis of the information complexity involved in determining winners in dynamic settings remains a promising avenue for future research, with potential implications for both theory and real-world decision-making systems.

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