How Many Votes Is a Lie Worth? Measuring Strategyproofness through Resource Augmentation

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ABSTRACT

Background: It is well known, by the Gibbard–Satterthwaite Theorem, that when there are more than two candidates, *any* non-dictatorial voting rule can be manipulated by untruthful voters.
Objectives and Research Questions: We aim to quantify *how strong* the incentive to manipulate is under different voting rules.
Methods: We suggest measuring the potential advantage of a strategic voter by asking how many copies of their (truthful) vote must be added to the election in order to achieve an outcome as good as their best manipulation. Intuitively, this definition quantifies what a voter can gain by manipulating in comparison to what they would have gained by finding like-minded voters to join the election. The higher the former is, the more incentive a voter will have to manipulate, even when it is computationally costly.
Results: We analyze the manipulation potential of well-known classes of rules. We show that the positional scoring rule with the smallest potential is always either Borda (when voters outnumber candidates) or Plurality (otherwise). We then prove that any rule satisfying even a weak form of majority consistency cannot beat Plurality, and that Majoritarian Condorcet rules perform significantly worse. Consequently, Borda stands out as the only one whose potential does not grow with the number of the voters.
Conclusions: By establishing a clear separation between different rules in terms of manipulation potential, our work paves a way to reason about strategic incentives and to guide the search for rules that provide voters with minimal incentive to manipulate.

KEYWORDS

Voting, Elections, Truthfulness, Strategic Behavior, Mechanism Design, Resource Augmentation

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All voting rules are manipulable.
But some rules are more manipulable than others.

1 INTRODUCTION

Alice has found herself in a conundrum. It is election day in Wonderland, and she firmly wants to help her favorite candidate, the Mad Hatter, become the next mayor. A well-versed student in social choice theory, Alice knows that any reasonable voting rule is manipulable, meaning voting untruthfully may help reach her goal [58, 82]. So how should she vote?

For Alice, answering this question will not be easy. She is aware that voter manipulation is NP-hard for many voting rules [9, 10, 53], and she worries that she might not find her optimal vote before the election day is over. Further, Alice is unsure about how the other residents of Wonderland will vote, and fears that a misestimate may cause her manipulation to do her more harm than good. Indeed, for many voting rules, Alice cannot theoretically guarantee lying will be *safe* unless she knows a significant portion of other votes [61], and she is further troubled by empirical results demonstrating the perils of manipulating under uncertainty [47, 65, 89]. Even if Alice is generally aware of the preferences of her compatriots, she realizes there may be other forms of uncertainty, such as rapidly shifting preferences or not knowing who will actually show up to the election [70]. Worst of all, she is scared other voters might be strategizing themselves! Consequently, the intel Alice has on these voters may also have been cleverly handpicked by them to avoid being exploited [60].

In the midst of her despair, Alice remembers she has a secret weapon: a car. Many of her neighbors fully agree with her (true) preferences over the mayoral candidates, but they will not interrupt their tea time to go and vote—unless someone drives them. Filled with excitement, Alice decides to abandon any efforts to compute her optimal untruthful vote, and to instead drive her honest neighbors to vote.[1] After all, this alternative plan requires no computation, no information on the rest of the electorate and not even

[1]For a real-life comparison, consider transportation services by various Get out the Vote (GOTV) efforts.

on the voting rule being used.[2] However, just as she is about to reach for her car keys, an anxious question strikes Alice: *just how many neighbors will she have to drive, before she can be certain that abandoning her efforts to manipulate was worth it?*

1.1 How Many Votes Is a Lie Worth?

In this paper, we resolve Alice's conundrum. More precisely, we investigate the number of additional (truthful) copies a voter would need in order to produce a result as desirable to them as her best manipulation. For most natural voting rules, getting sufficiently many copies should eventually reach this goal, since these rules favor overwhelming majorities. However, the exact number of copies needed can be quite different for different rules, as the following example shows.

Example 1.1. Consider two voting rules that map voters' rankings over m candidates to a winner. In *Borda Count* (*BC*), each voter awards $m-j$ points to their j^{th}-ranked candidate, and the candidate with the most points wins. *Instant Runoff* voting (*IRV*), on the other hand, iteratively eliminates the candidate with the least number of voters ranking them top, and the last-remaining candidate wins. Given 4 candidates and $n \geq 9$ voters, say the voters' preferences over candidates are:

Voter 1	$c_1 \succ c_2 \succ c_3 \succ c_4$
Voter 2	$c_4 \succ c_1 \succ c_2 \succ c_3$
$\frac{n-1}{4}$ voters	$c_2 \succ c_3 \succ c_4 \succ c_1$
$\frac{n-1}{4}$ voters	$c_3 \succ c_4 \succ c_1 \succ c_2$
$\frac{n-3}{2}$ voters	$c_4 \succ c_3 \succ c_2 \succ c_1$

Using Instant Runoff, c_1 is eliminated first, followed by c_3, and finally c_2, making c_4 the winner. Similarly, under Borda Count, the scores of candidates c_1, c_2, c_3, and c_4 are $\frac{n+19}{4}$, $\frac{5n+3}{4}$, $\frac{9n-13}{4}$, and $\frac{9n-9}{4}$, respectively, so the winner is once again c_4. Suppose voter 1 misreports her preferences as $c_3 \succ' c_1 \succ' c_2 \succ' c_4$. Now, the winner under either rule becomes c_3, whom voter 1 indeed prefers to c_4. However, while adding just two more copies of voter 1's truthful vote to the original profile would have also made c_3 the Borda Count winner, this is not the case for Instant Runoff: with less than $\frac{n-5}{4}$ copies of voter 1's truthful vote, the *IRV* winner is still c_4. Even with more truthful copies, c_4 may still win, depending on how we break ties.[3] Voter 1 will need $n-1$ additional copies of her truthful vote (in which case they become a majority, and the winner is c_1) to get an outcome that she undoubtedly (*i.e.*, regardless of the tiebreaker) prefers to what she can achieve by misreporting.

As Example 1.1 demonstrates, the same untruthful vote can outweigh different numbers of truthful ones under different voting rules. To analyze this "exchange rate" between lies and copies for different rules, we define the ***manipulation potential*** of a voting rule as the number of truthful copies needed (in the worst case) to ensure a voter could not have reached a better outcome by misreporting (see Section 3 for a formal definition). We attach three interpretations to this metric:

(1) Get out the vote (Alice's conundrum): If the electorate consists of communities with identical preferences, any member can dedicate their resources to increase the voter participation from their community. Then the manipulation potential of a rule is the *minimum* cost of misreporting (in terms of the cost of convincing one more community member to vote) sufficient to disincentivize manipulation. If the manipulation cost (*e.g.*, computational, or for gathering information) is larger, voters are better off investing their resources in raising community turnout.

(2) Endogenous participation: Taking a dual approach to (1), we can think of voters representing communities, with their weight corresponding to the number of members. The community leader(s) can recommend the members to vote untruthfully, but this may come at the cost of losing some fraction of the community who is unwilling to misreport their preferences (see, for example, Australian parties being "punished" by their voter base after issuing tactical how-to-vote cards in Instant Runoff elections [1, 79]). The manipulation potential is the minimum reduction in voter weight to guarantee manipulating does more harm than good.

(3) Approximate strategyproofness: As we cannot achieve strategyproofness with a non-dictatorial rule [58, 82], it is of interest to study how close we can get to it. Establishing approximations based on cardinal utilities (*e.g.*, "being truthful guarantees α fraction of the utility from best manipulation") is difficult since we only have access to ordinal preferences. For instance, in Example 1.1, the improvement in voter 1's utility (after changing the winner from c_4 to c_3) is tremendous if she is almost indifferent between $\{c_1, c_2, c_3\}$, but really dislikes c_4. However, it may also be insignificant if she is almost indifferent between $\{c_3, c_4\}$ instead. Rather than a utility-based analysis, we thus turn to a *resource augmentation* approach, where the outcome of the "best-possible" solution (in this case, the optimal manipulation) is compared with another method equipped with additional resources (in this case, truth telling with "bonus" votes) [80]. Thus, a rule's manipulation potential captures how approximately strategyproof it is. Resource augmentation and similar bicriteria approach are frequently employed in mechanism design, from where we draw our inspiration (see Section 1.2).

Overall, the manipulation potential framework offers a principled method of measuring the (worst-case) advantage of a strategic voter over her (augmented) truthful self. By establishing a clear separation between different rules in terms of manipulability, our work paves the way for the search for rules that provide voters with minimal incentive to manipulate.

Contributions and organization. The rest of this paper is structured as follows: In Section 1.2, we discuss the relationship of our results to prior work. After introducing our model (Section 2), we formally define the manipulation potential of a voting rule using a property we call *k-augmentation strategyproofness*, which requires that a voter will always weakly prefer adding k copies of her truthful self to any possible manipulation (Section 3). With our toolkit complete, we initiate our main agenda of finding the manipulation potential of well-known voting rules in Section 4, starting with rules such as plurality, instant runoff, and Borda Count (Section 4.1). We then characterize the manipulation potential of *all* positional scoring rules (Section 4.2), and conclude that the member of this class with the lowest manipulation potential will always be either Borda Count (if the number of voters outweighs the number of

[2]Although Alice likely knows that in Wonderland, election winners are determined using Dodgson's rule [43].
[3]See Section 3 on how we deal with ties.

Class of rules	Rule	Lower	Upper	
Positional Scoring Rules, $(s_1, s_2, \ldots, s_m)$	Borda Count (BC)	$m-2$ (Theorem 4.4)		
	Any rule with $s_1 = s_2$	∞ (Theorem 4.6)		
	Any rule with $s_1 \neq s_2$	$\min\{BC, PL\}$ (Thm. 4.10)	See Thm. 4.7	
	Plurality (PL)	$\lceil \frac{n-1}{2} \rceil$ (Theorem 4.2)		Majority-consistent
Condorcet Extensions	Black's Rule (BL)	$n-1$ (Theorem 4.11)		Majority-consistent
	Maximin (MM)	$\approx \frac{(m-2)(n)}{m-1}$ (Theorem 4.12)		Majority-consistent
Any majoritarian majority-consistent rule		$n-2$ (Theorem 4.14)	$n-1$ (Claim 4.1)	Majority-consistent
Any majority-consistent rule		$\frac{n-1}{2}$ (Theorem 4.15)	$n-1$ (Claim 4.1)	Majority-consistent
Other	Instant Runoff (IRV)	$n-1$ (Theorem 4.3)		Majority-consistent
	Plurality with Runoff (PLR)	$n-1$ (Theorem 4.3)		Majority-consistent
	Pareto	∞ (Proposition B.3)		
	Omninomination	∞ (Proposition B.3)		

Table 1: Summary of results showing bounds on the manipulation potential of the rules and rule classes studied in this paper. For lower bounds, see the theorem/proposition statement for the assumptions on n & m.

candidates) or Plurality (vice versa). Next, we turn our attention to Condorcet-consistent rules (Section 4.3), the manipulation potential of which has a trivial upper bound of $n-1$. We show that this bound is basically tight for any majoritarian rule satisfying a weak form of majority-consistency (thus any majoritarian Condorcet extension). In a general impossibility result, we show that *any* rule satisfying this weaker majority-consistency (and thus any Condorcet rule) will have a manipulation potential as high as plurality, and therefore (in the many-voter regime) much higher than Borda Count, the manipulation potential of which does not grow with the number of votes. A summary of our results for various voting rules can be found in Table 1. We conclude in Section 5. Appendix A for future directions and open problems. Due to space constraints, all proofs are deferred to the Appendix B.

1.2 Related work

1.2.1 Resource Augmentation. Our method is inspired by a classical result in auction theory by Bulow and Klemperer [29]. Take a single-item auction involving bidders with i.i.d. valuations. Here, a mechanism designer trying to maximize revenue faces challenges analogous to a strategic voter: the optimal auction may not be simple and requires knowing the distribution of valuations. Further, the correct choice under one distribution can significantly hurt revenue under another [60]. Luckily for the designer, as Bulow and Klemperer show, running a second-price auction with $n+1$ bidders will generate as much revenue as the revenue-maximizing auction with n bidders. Much like a voter reporting her truthful preferences, a second-price auction is simple and requires no knowledge of preferences. Intuitively, the theorem shows that the auctioneer is better off finding an additional bidder than to try and learn the bidders' preferences and compute the optimal auction.

There is a large body of work that extends this result to various auction settings by asking *how many* additional bidders are needed so that running the "simplest" auction (*e.g.*, VCG) yields expected revenue at least as high as the optimal auction—*e.g.*, Beyhaghi and Weinberg [13], Brustle et al. [26, 27], Derakhshan et al. [42], Eden et al. [48], Ezra and Garbuz [50, 51], Liu and Psomas [68]. This measure is referred to as the *competition complexity*.

This same approach of comparing a simple algorithm to the optimal solution with less resources has been applied in other domains under the name *resource augmentation* (*cf.* Roughgarden [80] for an overview), including allocation problems [2, 28], selfish routing [56, 81], scheduling [6, 62], and online paging [83].

Despite the parallels, our model also exhibits several key differences from above works. For instance, the manipulator is no longer the mechanism designer, but one of the *participants* in the mechanism. Consequently, while the resource augmentation in auction theory typically corresponds to adding bidders (that do not share the designer's incentives, but still benefit him by participating), in our case the resource being augmented is the weight of the voter herself, via truthful copies.

1.2.2 Degree(s) of Manipulability. Following the seminal result by Gibbard [58] and Satterthwaite [82] showing that any nondictatorial and onto voting rule can be manipulated by voters (when $m > 2$), a rich literature in social choice has focused on measuring manipulability of different rules.

Some of these results have focused on the *cost* of manipulation under different rules, including computational cost [9, 10, 37, 39–41, 52, 53, 69, 74, 90] and the cost of obtaining sufficient information on other votes [8, 25, 35, 59, 61, 63, 72, 91]. Despite these obstacles, voters may still misreport their vote if the potential *gain* is sufficiently large, especially as complexity results do not rule out heuristic manipulation algorithms that work well in practice [38, 41]. While metrics such as *incentive ratio* can be used to study gains from manipulation in settings with cardinal utilities (*e.g.*, [11, 31–34, 67, 88]), this is not applicable in our model (see Interpretation (3) in Section 1.1).

Separately, a substantial body of work studies the *frequency* with which manipulations can occur. To test different rules under this degree of manipulability (also called the Nitzan-Kelly index due to Nitzan [73] and Kelly [64]), one can study the fraction of elections in which the rule can be manipulated by using empirical data [44, 45], fixed distributions of preferences [3, 46], analytical approaches [54, 55], or exhaustive search over small instances [4, 5]. Along with the frequency of manipulations, some of this work measures the expected values of metrics such as the minimum coalition size necessary for manipulation [30, 78] or the (ordinal) improvement of the outcome in the preferences of the manipulator [5, 85]. In this paper, we do not make any assumptions on how voters are distributed and instead focus on how far ahead of a truthful vote a manipulation can get (in terms of copies needed) in the worst case.

2 PRELIMINARIES

Election instances. Let $N \coloneqq \{1, 2, \dots, n\}$ be a finite set of *voters* and $C \coloneqq \{c_1, c_2, \dots, c_m\}$ be a finite set of *candidates* for some $n \geq 2$ and $m \geq 3$. We assume that each voter $i \in N$ has a strict *preference order* $\succ_i$, which is a transitive, antisymmetric, and complete relation over candidates C. We denote the set of all possible preference orders by $\mathcal{L}(C)$. For any voter $i \in N$ and any two candidates $c, c' \in C$, we denote $c \succ_i c'$ to indicate $(c, c') \in \succ_i$ (*i.e.*, voter i ranks c strictly above c') and $c' \succeq_i c$ otherwise (*i.e.*, either $c' \succ_i c$ or $c' = c$). A *preference profile* $\succ_N \coloneqq (\succ_i)_{i \in N} \in \mathcal{L}(C)^n$ is a vector containing the preference orders of all voters. Given a profile $\succ_N$, a voter $i \in N$, and an alternative preference order $\succ'_i \in \mathcal{L}(C)$, we use $(\succ'_i, \succ_{-i})$ to denote the preference profile where voter i's preference order is replaced by $\succ'_i$ and all other voters' preferences remain unchanged, *i.e.*, $(\succ'_i, \succ_{-i}) \coloneqq (\succ_1, \dots, \succ_{i-1}, \succ'_i, \succ_{i+1}, \dots, \succ_n)$.

We sometimes refer to $\succ'_i$ as a *manipulation* by voter i, as opposed to her *truthful vote* $\succ_i$. For any profile $\succ_N$, voter $i \in N$, and integer $k \geq 0$, we use $\succ_N + k(\succ_i)$ to denote $\succ_N$ with k additional copies of voter i's preference order $\succ_i$, *i.e.*,

$$\succ_N + k(\succ_i) \coloneqq (\succ_1, \dots, \succ_n, \underbrace{\succ_i, \dots, \succ_i}_{k \text{ times}})$$

so that $\succ_N + k(\succ_i)$ is a profile with $n + k$ voters rather than n.

Manipulability. A *social choice function (SCF)* is a mapping $f : \mathcal{L}(C)^* \to C$ from each preference profile to a unique[4] *winner* among the candidates C.[5] Given a profile $\succ_N$ and an SCF f, a manipulation $\succ'_i$ is *profitable* for voter i if reporting it results in an outcome she strictly prefers, *i.e.*,

$$f(\succ'_i, \succ_{-i}) \succ_i f(\succ_N).$$

An SCF is *strategy-proof* if it admits no profitable manipulations, and it is *manipulable* otherwise.

[4]In Section 3, we explain how we incorporate set-valued functions (that might return ties) into our framework.

[5]In this paper, we only consider SCFs that are defined on all profiles with *any* number of voters.

3 AUGMENTATION STRATEGYPROOFNESS AND THE MANIPULATION POTENTIAL

In this section, we will introduce the central metric of our paper, the *manipulation potential* of social choice functions. To do so, we first extend the strategyproofness definition given in Section 2.

Definition 3.1 (k-Augmentation Strategyproof). Let k be a non-negative integer. A social choice function f is *k-augmentation strategyproof (k-ASP)* if for any profile $\succ_N \in \mathcal{L}(C)$, voter $i \in N$, and alternative preference order $\succ'_i \in \mathcal{L}(C)$, we have $f(\succ_N + k(\succ_i)) \succeq_i f(\succ'_i, \succ_{-i})$.

In words, an SCF is k-ASP if a voter is never strictly better off misreporting her vote rather than having k copies of her truthful self join the election. This indeed generalizes standard strategyproofness, which is equivalent to 0-ASP.

Definition 3.2 (Manipulation Potential). For any SCF f, we define its *manipulation potential* as

$$\texttt{MP}(f) \coloneqq \min\{k \in \mathbb{Z} : f \text{ is } k'\text{-ASP for all } k' \geq k\}$$

and $\texttt{MP}(f) \coloneqq \infty$ if no such k exists.

In words, $\texttt{MP}(f)$ is the minimum number of additional copies a voter would need in order to ensure that she prefers the outcome with her copies (even if more copies join) to that of any manipulation she could do in the original profile. From a resource augmentation perspective, an SCF with low manipulation potential limits the relative impact of a manipulation, because a voter can achieve the same impact by telling the truth with a few additional resources (*i.e.*, votes).

Non-monotonicities. A social choice function f that is k-ASP can still have $\texttt{MP}(f) > k$, since it may not be k'-ASP for some $k' > k$. While this may seem unnatural, adding copies of voter's truthful vote may initially *hurt* her in certain profiles under many natural SCFs.[6] Some profiles may exhibit even more layers of non-monotonicities, as the next example shows.

Example 3.3. Consider a profile $\succ_N$ with $n = 27$ voters:

Voter 1	$c_1 \succ_1 c_2 \succ_1 c_3 \succ_1 c_4 \succ_1 c_5$
3 voters	$c_1 \succ_i c_3 \succ_i c_4 \succ_i c_2 \succ_i c_5$
3 voters	$c_3 \succ_i c_5 \succ_i c_4 \succ_i c_2 \succ_i c_1$
5 voters	$c_4 \succ_i c_3 \succ_i c_2 \succ_i c_5 \succ_i c_1$
6 voters	$c_2 \succ_i c_3 \succ_i c_4 \succ_i c_5 \succ_i c_1$
9 voters	$c_5 \succ_i c_3 \succ_i c_4 \succ_i c_2 \succ_i c_1$

If we run Instant Runoff (*IRV*, as defined in Example 1.1) on $\succ_N$, it will eliminate c_3 first, followed by c_1, c_2, and c_5, making c_4 the winner. If voter 1 instead reported the preference order $c_3 \succ'_1 c_1 \succ'_1 c_2 \succ'_1 c_4 \succ'_1 c_5$, she can ensure $IRV(\succ'_1, \succ_{-1}) = c_3$, which is a better outcome for her. As seen from Figure 1, on the other hand, adding gradually more copies of voter 1 to the original profile will initially result in a better outcome than the manipulation, followed by the worst possible outcome for her, before eventually securing a better outcome than manipulating even if more copies join.

[6]This is closely related to the *no-show paradox*, where a voter would be better off if they had not voted; *cf.* Moulin [71].

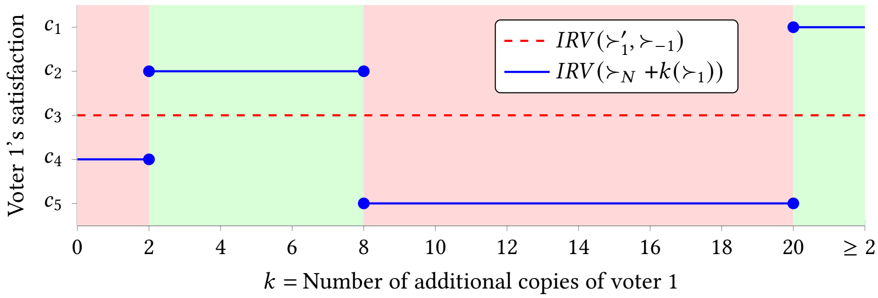

Figure 1: For the profile from Example 3.3, the *IRV* winners of $\succ_N + k(\succ_1)$ and $(\succ'_1, \succ_{-1})$ as a function of k. Green regions indicate the values of k for which voter 1 prefers the outcome of the former to the latter.

An alternative definition for manipulation potential would have been the minimum k for which a rule is k-ASP, regardless of its behavior for any $k' > k$. However, this type of resource augmentation would suffer from miscoordination issues similar to strategic voting, namely that overshooting can result in undesirable outcomes [84]. As our goal is to benchmark manipulation against the (resource augmented version of) a simple strategy, Def. 3.2 leaves no room for such miscoordination. In any case, as we will see, whenever we prove an SCF f has $\texttt{MP}(f) \geq k$ for some k, we will also show that f is not k'-ASP for *any* $k' < k$, so the result would not have changed under either definition.[7] This shows that while non-monotonicities can arise in specific profiles like Example 3.3, they generally do not affect the manipulation potential in the worst case.

Tie(breaker)s. Many of rules we consider in later sections are in fact *social choice correspondences (SCCs)* of form $F : C \to \mathcal{P}(C)$, where $\mathcal{P}(C)$ is the power set of C. In words, an SCC may return multiple winning candidates at once; in this case, we say the winners are *tied*. To extend Definitions 3.1 and 3.2 to a social choice correspondence F, we equip it with a *linear tie-breaker* $\succ_T \in \mathcal{L}(C)$ and define F_T as the social choice function that maps a profile $\succ_N$ to the top choice of $\succ_T$ in $F(\succ_N)$, *i.e.*,

$$F_T(\succ_N) \coloneqq \max_{\succ_T} F(\succ_N).^8$$

We then say an SCC F is k-ASP if F_T is k-ASP *for all* $\succ_T \in \mathcal{L}(C)$, so that we have

$$\texttt{MP}(F) \coloneqq \max_{\succ_T \in \mathcal{L}(C)} \texttt{MP}(F_T).$$

In fact, the manipulation potential of the SCCs we consider will be independent of the specific tiebreaking order $\succ_T$, since they all satisfy *neutrality*, *i.e.*, their outputs are invariant under relabeling the candidates. Formally, given a candidate permutation $\pi : C \to C$ and profile $\succ_N$, define $\pi(\succ_N)$ as the profile $\succ'_N$ where for any voter $i \in N$ and candidate pair $c, c' \in C$, we have $c \succ_i c'$ if and only if $\pi(c) \succ'_i \pi(c')$. Then, an SCC F is *neutral* if for any candidate permutation π, profile $\succ_N$, and candidate $c \in C$, we have $c \in F(\succ_N)$ if and only if $\pi(c) \in F(\pi(\succ_N))$. It is worth noting that even if the set-valued SCC F is neutral, the single-valued SCF F_T will not be, since $\succ_T$ will prioritize some candidates over others. For simplicity, we sometimes slightly abuse terminology and refer to both SCCs and SCFs f as "rules", and whether they are single- or multi-valued will be clear from context.

Additionally, all of the SCCs we consider satisfy *anonymity*, *i.e.*, their outputs are robust to relabeling the voters. Formally, given a voter permutation $\tau : N \to N$ and a profile $\succ_N$, define $\tau(\succ_N)$ as the profile $\succ'_N$ where for any voter $i \in N$ and candidate pair $c, c' \in C$, we have $c \succ_i c'$ if and only if $c \succ'_{\tau(i)} c'$. Then, an SCC F is *anonymous* if for any voter permutation τ and profile $\succ_N$, we have $F(\succ_N) = F(\tau(\succ_N))$. With anonymous rules, only the number of times each vote is cast matters, and not the identities of the voters that cast them. This property will be helpful when dealing with varying number of total voters (going from $\succ_N$ to $\succ_N + k(\succ_1)$).

4 COMPUTING THE MANIPULATION POTENTIAL OF VOTING RULES

In this section, we report our results on the manipulation potential of known social choice correspondences, which are summarized in Table 1. For each SCC F, the proofs consist of establishing upper and lower bounds for $\texttt{MP}(F)$. For clarity, we outline below the general structure of our proofs.

Proof structure. Leveraging anonymity and neutrality of the SCCs we consider (Section 3), we will focus, without loss of generality, on manipulations by voter $1 \in N$ and assume that her true preferences are $c_1 \succ_1 c_2 \succ_1 \ldots \succ_1 c_m$. To show that a given social choice correspondence F has a manipulation potential of strictly more than k—*i.e.*, $\texttt{MP}(F) > k$—it will suffice to provide a specific profile $\succ_N$, manipulation $\succ'_1$, and tiebreaker $\succ_T$ such that $F_T(\succ'_1, \succ_{-1}) \succ_1 F_T(\succ_N + k(\succ_1))$, which shows F is not k-ASP. For constructing these profiles, it will sometimes be helpful to assume a (constant) lower bound or divisibility requirement on n or m; these will be clearly stated in the theorem statements. For proving upper bounds—*i.e.*, $\texttt{MP}(F) \leq k$—we will have to prove that *for any* $\succ_N, \succ'_1, \succ_T$, and $k' \geq k$, we have $F_T(\succ_N + k'(\succ_1)) \succeq_1 F_T(\succ'_1, \succ_{-1})$.

For example, an SCC F is *majority-consistent* if whenever a candidate $c \in C$ is the top choice of a strict majority of voters in $\succ_N$, we have $F(\succ_N) = \{c\}$. For any majority-consistent rule, adding at least $n - 1$ copies of a voter will guarantee that her most-preferred candidate wins. Since no manipulation can lead to a better result than this, the upper bound criterion given above is satisfied.

[7] The one exception to this is our impossibility result for biranking-majority-consistent rules presented in Theorem 4.15

[8] When breaking ties as so, k-ASP for SCCs is a generalization of *strong Fishburn-strategyproofness* (with equivalence at $k = 0$). For an excellent overview of different tiebreakers and corresponding manipulability notions, see Brandt et al. [24].

Claim 4.1. *The manipulation potential of any majority-consistent rule is at most $n-1$.*

Many, but not all, rules we consider are majority-consistent (for a non-example, consider Borda Count). As we show next, for some, we can achieve a tighter upper bound than that of Claim 4.1.

4.1 Plurality (w/ Runoff), Instant Runoff, Borda

We first analyze the manipulation potential of four example rules.

4.1.1 Plurality. Under Plurality (*PL*), the winner is the candidate that is ranked top by the most number of voters (ties broken according to the tiebreaker $\succ_T$). While plurality is majority consistent, as we show next, its manipulation potential is roughly half of the upper bound in Claim 4.1.

Theorem 4.2. *The manipulation potential of plurality is* $\mathsf{MP}(PL) = \lceil \frac{n-1}{2} \rceil$. *Further, PL is not k-ASP for any* $0 \leq k < \lceil \frac{n-1}{2} \rceil$.

The proof can be found in Section B.1. Recall from Definition 3.2 that to prove $\mathsf{MP}(PL) \geq \mathsf{MP}(PL) = \lceil \frac{n-1}{2} \rceil$, it would have been sufficient to show *PL* is not k-ASP for $k = \mathsf{MP}(PL) = \lceil \frac{n-1}{2} \rceil - 1$. Hence, the second part of the theorem statement is indeed a stronger claim (*cf.* note on non-monotonicities in Section 3).

That plurality beats the upper bound in Claim 4.1 brings about a natural question: can we find a tighter bound for the manipulation potential of all majority consistent rules? To answer this question, we next consider adding *runoff rounds* to Plurality.

4.1.2 Plurality with Runoff & Instant Runoff. Just like with Plurality, under Plurality with Runoff (*PLR*), each voter gives one point to her top-ranked candidate in the first round, but now the *two* candidates with the highest scores advance to the runoff (*i.e.*, the second round). In the runoff, each voter gives one point to her preferred candidate among the two, and the candidate with the higher score wins. With Instant Runoff (*IRV*),[9] on the other hand, the winner is determined by iteratively eliminating the candidate with the least top-choice votes from the profile, as explained in Example 1.1. This continues until only one candidate remains, who is declared the winner.

For both rules, we define their set-valued output (prior to tiebreaking) using *parallel-universes tiebreaking* [36], which dictates that a candidate c is one of the winners (*i.e.*, $c \in PLR(\succ_N)$ or $c \in IRV(\succ_N)$) if there exists *some* way to break ties (when deciding who advances to the next round of the runoff) such that c wins. As with other SCCs, tiebreaker $\succ_T$ then resolves the tie in favor of its highest-ranked candidate to determine PLR_T or IRV_T.

Both *PLR* and *IRV* are majority-consistent. Further, as we show next, their manipulation potential is exactly $n-1$, showing that the upper bound in Claim 4.1 is the tightest we can get for this class.

Theorem 4.3. *The manipulation potentials of Plurality with Runoff & Instant Runoff are* $\mathsf{MP}(PLR) = \mathsf{MP}(IRV) = n-1$ *(lower bound assumes* $m \geq 4$, $n \geq 5$ *and that* $n-1$ *is divisible by 4). Further, neither rule is k-ASP for any* $0 \leq k < n-1$.

All three of the rules so far (PL, PLR, IRV) have manipulation potentials growing linearly with the number of voters n, even if the coefficient of growth for *PL* is half as big. Can there be a rule whose manipulation potential *does not* grow with n? We next answer this question in the affirmative.

4.1.3 Borda Count. As introduced in Example 1.1, under Borda Count (*BC*), each voter contributes $m-j$ points to her j^{th}-ranked candidate for $j \in \{1, 2, \ldots, m\}$ and the candidate with the highest total "Borda score" wins. As we show next, unlike the previous rules, the manipulation potential of *BC* grows linearly with the number of candidates and is independent of the number of voters.

Theorem 4.4. *The manipulation potential of Borda Count is* $\mathsf{MP}(BC) = m-2$ *(lower bound assumes n is even). Further, BC is not k-ASP for any* $0 \leq k < m-2$.

Our analysis of the rules of this subsection (PL, PLR, IRV, BC) already displays that the "effective weight" of a single manipulation (in terms of truthful copies needed to create the same impact) varies significantly between rules (Figure 2). In the next two subsections, we shift our focus to characterizations and impossibility results on the manipulation potentials of broad rule *classes*.

4.2 All Positional Scoring Rules

Out of the rules studied so far, the lower manipulation potential belongs to either Plurality (if $\lceil \frac{n-1}{2} \rceil < m-2$) or Borda Count (vice versa). As we will show, this in fact remains the case for much broader classes of rules. To formalize this claim, we first introduce a class that includes *PL* and *BC*.

Definition 4.5 (Positional Scoring Rules). A (monotonic) *positional scoring rule* is a SCC F^s associated to a vector $s := (s_j)_{j \in \{1,\ldots,m\}}$ with $1 = s_1 \geq s_2 \geq \ldots \geq s_m = 0$. On input profile $\succ_N$, the output of F^s is computed as follows: each voter i contributes s_ℓ points to their ℓ^{th}-ranked candidate in $\succ_i$, and F^s returns the candidate(s) with highest total score.

PL and *BC* correspond to the positional scoring rules associated with vectors $s = (1, 0, \ldots, 0)$ and $s = (1, \frac{m-2}{m-1}, \frac{m-2}{m-1}, \ldots, \frac{1}{m-1}, 0)$, respectively (note that scaling all the scores by $\frac{1}{m-1}$ for Borda Count does not change the output). Our first result indicates that unlike these two rules, the manipulation potential of some positional scoring rules can be unbounded!

Theorem 4.6. *For any* F^s *with* $s_1 = s_2$, *we have* $\mathsf{MP}(F^s) = \infty$.

In words, if a positional scoring rule treats the top two positions equally, adding more copies of a voter cannot help her top choice gain an edge over her second top choice, while misreporting can.

How about when $s_1 > s_2$? Ideally, we would like to generalize our approach to *PL* (Theorem 4.2) and *BC* (Theorem 4.4). To do so, we introduce some additional notation. For $\ell = 1, \ldots, m$, we define

$$\overline{s}(\ell) := \frac{1}{\ell} \sum_{\ell'=1}^{\ell} s_{\ell'} \quad \text{and} \quad \underline{s}(\ell) := \frac{1}{\ell} \sum_{\ell'=m-\ell+1}^{m} s_{\ell'}$$

as the the average score assigned to the top and bottom ℓ positions, respectively. Further, for $\ell = 2, 3, \ldots, m-1$, we denote $\alpha^s_\ell := \max\{\ell' \in \{1, ..., \ell-1\} : s_{\ell'} > s_{\ell+1}\}$, which is well-defined since $s_1 > s_2 \geq s_{\ell+1}$. For example, for Plurality, $\overline{s}(\ell) = \frac{1}{\ell}$; $\underline{s}(\ell) = \frac{1}{m}$ if $\ell = m$ and 0 otherwise; and $\alpha^s_\ell = 1$. With our notation in place, we now present a general upper bound for positional scoring rules.

[9]Also known as Single Transferable Vote (STV).

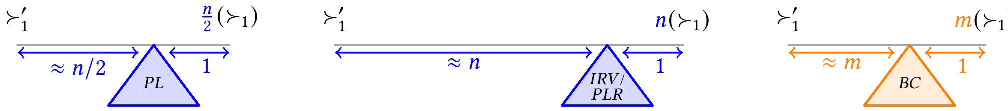

Figure 2: Visualization of our results from Section 4.1. For each rule, the longer the left arm of the scale, the more truthful copies a single manipulation can "lift". That is, the voter will need (in the worst case) a larger number of truthful copies to produce an outcome as desirable as that of the manipulation. The left arm of the middle scale is double that of the left scale, whereas comparing it to the rightmost scale depends on n and m.

Theorem 4.7. *For any F^s with $s_1 \neq s_2$, we have that $\mathsf{MP}(F^s) \leq \max_{\ell \in \{1,2,\ldots,m-1\}} U_s(\ell)$, where*

$$U_s(\ell) := \begin{cases} \frac{1}{s_\ell - s_{\ell+1}} - 1 & \text{if } \ell = 1 \\ \min\left(\frac{1}{s_\ell - s_{\ell+1}} - 1, \frac{(n-1)(\bar{s}(2) - \underline{s}(\alpha^s_\ell)) + \frac{1}{2}}{\bar{s}(\alpha^s_\ell) - s_{\ell+1}}\right) & \text{otherwise} \end{cases}.$$

We interpret $\frac{1}{s_\ell - s_{\ell+1}} = \infty$ whenever $s_\ell = s_{\ell+1}$. Since $s_1 > s_2$, we have $U_s(1) < \infty$. For all other $\ell \in \{2, \ldots, m-1\}$, we have $\bar{s}(\alpha^s_\ell) > s_{\ell+1}$ by definition of α^s_ℓ, so $U_s(\ell) < \infty$, as the second entry of the min is finite. Hence, Theorem 4.7 gives a finite upper bound on $\mathsf{MP}(F^s)$, showing that the sufficent condition in Theorem 4.6 ($s_1 = s_2$) is also necessary for an unbounded manipulation potential. For *PL* and *BC*, this upper bound corresponds to $\frac{n}{2}$ and $m-2$, respectively, hence Thm. 4.7 indeed generalizes the upper bounds from Theorems 4.2 and 4.4 (with a minor loss for *PL* if n is odd).

Remark 4.8. Theorem 4.7 also shows that having a manipulation potential that does not grow with n is not unique to Borda Count. In particular, whenever $s_i \neq s_j$ for any $i \neq j$, $\mathsf{MP}(F^s)$ will eventually stop growing with n due to the first term inside the min in each $U_s(\ell)$. Still, out of any such rule, Borda Count is the one with the lowest manipulation potential, as we show below (Theorem 4.10).

We next present a general lower bound on the manipulation potential of positional scoring rules.

Proposition 4.9. *For any F^s with $s_1 \neq s_2$, we have $\mathsf{MP}(F^s) \geq \max_{\ell \in \{1,2,\ldots,m-1\}} L_s(\ell)$, where*

$$L_s(\ell) := \min\left(\frac{1}{s_\ell - s_{\ell+1}} - 1, \min_{t \in \{1,2,\ldots,\alpha^s_\ell\}} \left\lfloor \frac{(n-2)(\bar{s}(2) - s_{m-t+1}) + 1 - s_{m-t}}{s_t - s_{\ell+1}} \right\rfloor\right),$$

assuming n is even and at least $\frac{2s_2}{1-s_2}$. Further, F^s is not k-ASP for any $k < \max_{\ell \in \{1,2,\ldots,m-1\}} L_s(\ell)$.

It is worth pointing out two shortcomings of Proposition 4.9: First, the second term inside the min in $P_s(\ell)$ does not exactly match the one inside $U_s(\ell)$ from Theorem 4.7. Second, unlike the minimal lower bound / divisibility requirements we have been making on n or m so far, requiring $n \geq \frac{2s_2}{1-s_2}$ can significantly limit the "feasible region" of Proposition 4.9 for certain rules F. For example if $s_2 \geq \frac{m-2}{m-1}$ (which is satisfied by *BC*), which then requires $n \geq 2(m-2)$, whereas we would ideally like to find the positional scoring rule(s) with the lowest manipulation potential regardless of whether we have more voters or candidates. Both of these limitations follow from the fact that constructing the "worst case" profile described in the proof sketch of Theorem 4.7 is more difficult for some positional scoring rules than others, and impossible for yet others (for example, requiring every candidate in $\{c_1, c_2, \ldots, c_{\alpha^s_\ell}\}$ reaching the score of $c_{\ell+1}$ after the same number of turns might require some of them to start from negative scores in the original profile). Overcoming these difficulties to get a tighter bound may require much more complex and unnatural restrictions on n, m, and the scoring cost s. In any case, as we show next, the lower bound established in Prop. 4.9 is sufficient for our main goal in this subsection, which is to prove that the positional scoring rule with the smallest manipulation potential will always be either Plurality or Borda Score, without any assumptions on whether there are more voters or candidates.

Theorem 4.10. *For any positional scoring rule $F^s \notin \{PL, BC\}$, given some $n \geq 4$ that is divisible by 4, we have either $\mathsf{MP}(F^s) > \mathsf{MP}(BC)$ or $\mathsf{MP}(F^s) \geq \mathsf{MP}(PL)$. Further, if $n \geq \max(6, \frac{2}{s_2} + 4)$, then either $\mathsf{MP}(F^s) > \mathsf{MP}(BC)$ or $\mathsf{MP}(F^s) > \mathsf{MP}(PL)$* (i.e., *the second inequality is also strict).*

Having shown either Plurality or Borda Count (depending on n and m) has the lowest manipulation potential of an (infinitely sized) class of rules they belong to, we next move on to another class of rules, this time containing neither *PL* nor *BC*, and show that, once again, any of its members will be defeated by one or both of these rules in terms of their manipulation potential.

4.3 Condorcet (& Birank-Majority) Consistency

In this section, we study the manipulation potential of Condorcet consistent rules. After introducing Condorcet consistency, we first analyze two example rules. We then prove two general impossibility results on rules satisfying a weaker form of majority-consistency (hence any Condorcet extension). First, any such rule that determines the winner solely on pairwise defeats (the so-called *majoritarian* rules) will have a manipulation potential of at least $n-2$. Second, when n is odd, the manipulation potential of **any rule satisfying this weaker form of majority consistency (and thus of any Condorcet extension)** is as high as that of Plurality. As a consequence, the manipulation potential of Borda Count is significantly lower than any of these rules whenever $n >> m$.

Condorcet consistency. A *Condorcet winner* is a candidate who pairwise defeats every other candidate. Formally, for a profile $\succ_N$, its *margin matrix* $M(\succ_N)$ is a $m \times m$ matrix with its entry corresponding to candidates c, c' equal to $M(\succ_N)[c, c'] := |\{i \in N : c \succ_i c'\}| - |\{i \in N : c' \succ_i c\}|$. Then c is the Condorcet winner if $M(\succ_N)[c, c'] > 0$ for all $c' \in C \setminus \{c\}$. A SCC/SCF is *Condorcet-consistent* (or a *Condorcet extension*) if it only picks the Condorcet winner whenever one exists.

Any Condorcet extension is majority-consistent, since a majority winner is also a Condorcet winner. Consequently, the upper bound of $n-1$ from Claim 4.1 also applies to Condorcet extensions. On the other hand, not every majority-consistent rule is a Condorcet extension, *e.g.*, *PL*, *IRV*, and *PLR* all fail Condorcet consistency. We now present two example Condorcet extensions.

4.3.1 Black's Rule. As we have seen in Section 4.1, Borda Count (*BC*) stands out among the rules we have studied by having a manipulation potential that does not grow with the number of voters, which, in practice, can be significantly larger than the number of alternatives. However, *BC* fails even majority-consistency. Could there be a Condorcet extension that shares the same manipulation potential as *BC*? A natural candidate is Black's Rule (*BL*), which is simply defined as follows: if there is a Condorcet winner, output that. Otherwise, output the Borda Count winner(s). However, as we show next, *BL* has a manipulation potential as large as any Condorcet extension can have.

Theorem 4.11. *The manipulation potential of Black's Rule is* $\mathsf{MP}(BL) = n-1$ *(lower bound assumes* $m \geq 4$ *and odd* n*). Further, BL is not* k*-ASP for any* $0 \leq k < n-1$.

As Theorem 4.11 shows, adding the conditional on Condorcet winner can be sufficient to go from a manipulation potential that grows linearly with m (Borda Count) to one that grows linearly with n instead (Black's Rule). Every rule we have studied so far has fallen exclusively to one of these two categories (except for the general characterization of positional scoring rules in Section 4.2). To show this is not always the case, we next present a Condorcet extension whose manipulation potential depends on both m and n.

4.3.2 Maximin. The SCC known as Maximin[10] is defined using the margin matrix $M(\succ_N)$ as

$$MM(\succ_N) = \arg\max_{c \in C} \min_{c' \in C} M(\succ_N)[c, c'].$$

In words, Maximin scores candidates according to their lowest margin against other candidates and then outputs the candidate(s) with the highest score.

Theorem 4.12. *The manipulation potential of Maximin is at most* $\frac{(m-2)n}{m-1}+2$ *and (assuming* $n-2$ *is divisible by* m*) at least* $\frac{(m-2)(n-2)}{m-1}+1$*. Further, MM is not* k*-ASP for any* $k < \frac{(m-2)(n-2)}{m-1}+1$.

4.3.3 Impossibilities. Both of the example Condorcet extensions above have a manipulation potential that is always larger than that of Plurality (for MM, note $\frac{m-2}{m-1} \geq \frac{1}{2}$) and thus larger than that of Borda Count whenever $n \geq 2m$. To study this trend more broadly, we now turn our attention to general impossibility results. The main question we seek to answer is this subsection is: *Can there be a Condorcet extension with a lower manipulation potential than (both) Plurality and Borda Count?* The answer, as we will show, is no. To strengthen our negative results, we first significantly weaken Condorcet consistency, to a property that is even weaker than majority consistency.

Definition 4.13 (Biranking-Majority-Consistent). Consider a profile consisting of only two types of rankings, *i.e.*, there exists $\succ_a, \succ_b \in \mathcal{L}(C)$ such that for any voter $i \in N$, we have $\succ_i \in \{\succ_a, \succ_b\}$. We say SCC/SCF is *biranking-majority-consistent* if for any such profile where one of the rankings appears strictly more times than the other, it picks the top choice of the more frequent ranking.

Majority consistency (and therefore Condorcet consistency) indeed implies biranking majority consistency. To see the reverse is false, consider Coombs' Rule, which is defined analogously to Instant Runoff (*IRV*) except in each round, instead of the candidate with the least top-choice votes, the candidate with the most bottom-choice votes is eliminated. Using this rule, even if a majority of voters agree on their top-choice candidate, this candidate can get eliminated if the same group of voters disagree about their bottom choices; however, this will not happen if they all submit the same ranking. Hence, Coombs' Rule is biranking-majority-consistent but not majority-consistent.

Our first impossibility result focuses on so-called *majoritarian* rules, which decide on a winner solely based on the majority defeats between pairs of candidates. Formally, a SCC F is majoritarian if for any two profiles $\succ_N, \succ'_N$ such that $\operatorname{sign}(M(\succ_N)[c,c']) = \operatorname{sign}(M(\succ'_N)[c,c'])$ for any two candidates $c, c' \in C$, then we have $F(\succ_N) = F(\succ'_N)$. Our first impossibility results shows that majoritarian biranking-majority-consistent rules (and thus majoritarian Condorcet extensions) cannot get much better than the worst-case for Condorcet extensions given in Claim 4.1.

Theorem 4.14. *Any SCC F that is (1) neutral (2) biranking-majority-consistent and (3) majoritarian will have* $\mathsf{MP}(F) \geq n-2$*. Further, F is not* k*-ASP for any* $k < n-2$.

Can we hope for a much lower manipulation potential if we remove the majoritarian requirement? After all, Plurality satisfies (biranking) majority consistency, and outperforms the lower bound of Thm. 4.14 by a factor of (roughly) $\frac{1}{2}$. As our next and last theorem shows, this is essentially the best we can achieve with *any* biranking-majority-consistent (thus any Condorcet-consistent) rule.

Theorem 4.15. *Whenever n is odd, any biranking-majority-consistent SCF* f *has* $\mathsf{MP}(f) \geq \frac{n-1}{2}$.

Since a biranking-majority-consistent SCC equipped with any tiebreaker is a biranking-majority-consistent SCF, Theorem 4.15 also applies to multi-valued rules (*cf.* discussion on ties in Section 3).

5 CONCLUSION

When confronted with the manipulability of his voting method, 18th century mathematician Jean-Charles de Borda famously claimed "My scheme is only intended for honest men" [14]. As we show in this paper, his rule can also tolerate indecisive manipulators with a few honest friends, while the situation can be much grimmer for the class of rules named after his intellectual rival Marquis de Condorcet (at least for large electorates). Just how many of these honest copies a voter needs before abandoning manipulation under different models and rules is an exciting research agenda that can significantly enhance our understanding of manipulation under limited resources.

[10] Maximin is also referred to as minimax (as it minimizes the worst defeat) or the Simpson-Kramer rule [20].

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A FUTURE DIRECTIONS AND OPEN PROBLEMS

In this work, we proposed to measure the comparative advantage of a strategic voter from a resource augmentation perspective, asking how many additional truthful copies of her vote would suffice to ensure an outcome at least as good as her best manipulation. This novel approach to manipulability gives rise to natural future directions and open problems, some of which we discuss below.

(a) Beating Borda. As we have seen in Section 4, in elections with many voters, the manipulation potential of Borda Count is significantly lower than any other rule we have studied. While Theorems 4.10 and 4.15 rule out two wide classes of voting rules from defeating Borda in this regime, a natural next question is to investigate how far this result generalizes. On the positive side, can we identify voting rules—either from prior literature or by designing new ones—with n-independent manipulation potentials that outperform Borda? On the negative side, can we identify some minimal properties that are sufficient for further impossibility results showing this will not happen?

(b) Automating the search. A promising avenue to investigate (a) and related open problems is to adopt an *automated reasoning* approach, *i.e.*, to use computer-generated proofs to gain intuition on small instances, and then generalize these via theoretical analysis. Indeed, after Tang and Lin [86, 87] demonstrated the suitability of this approach to social choice by reproving the Gibbard–Satterthwaite impossibility theorem (as well as the impossibility theorem of Arrow [7]), tools such as SAT solvers have paved the way to many new and important results [16, 17, 21, 22, 49, 57, 71]. Similarly, one can use such tools to search for rules with low manipulation potential, or for proofs strengthening Gibbard–Satterthwaite (which already proves no nondictatorial rule can have $\mathtt{MP}(f) = 0$) to give stronger lower bounds. A key challenge is that manipulation potentials inherently deal with elections with a varying number of voters, which may be difficult to encode with SAT solvers and, even if encoded, can lead to unnatural rules that behave differently under different n. A potential solution to this obstacle is to strengthen our anonymity requirement (Section 3) to *homogeneous rules*, for which only the frequency of each ranking matters. By encoding these frequencies, one can seek to minimize manipulation potential via, for example, SMT Solvers [15] or (mixed integer) linear programs [12, 76].

(c) Beyond single-winner. While our work focuses on analyzing the manipulation potential of rules that map voters' rankings to a single winner (or a tie, later to be broken), our framework can be easily adapted to rules with other input formats (*e.g.*, ratings/scores, approval ballots) and desired outputs (an aggregate ranking, a committee of winners, or a probability distribution over candidates). As a concrete example, take approval-based multiwinner elections, where the goal is to output a committee of candidates. Here, even weak forms of proportionality are incompatible with strategyproofness [75]. A natural question is to investigate the implication of this result on the manipulation potential of multiwinner rules. Can we identify or design rules with strong proportionality guarantees (*e.g.*, see properties such as JR, EJR, EJR+, or core stability [66]) *and* low manipulation potential? Or does assuming such properties impose strong lower bounds on the manipulation potential? What is the manipulation of known multiwinner rules, such as Proportional Approval Voting, or the Method of Equal Shares [77]? We consider such extensions to be valuable next steps for the study of manipulation potentials.

(d) Budgeted manipulations. One can view the manipulation potential as the *exchange rate* between a single manipulation and adding truthful copies. Given a rule with manipulation potential k, a voter is better off finding k copies of herself if the total cost to do this will be less than the cost of manipulation (see Interpretation (1) in Section 1.1). An interesting follow-up is to consider whether a voter can benefit from *combining* these two strategies (manipulating and finding copies). More concretely, consider a model that specifies the cost a voter must incur to find her (truthful) copies, as well as the cost manipulations of different "magnitude" (*e.g.*., the cost can be increasing with the swap-distance between her true vote and the untruthful one). If the voter has a fixed budget to begin with, what is her optimal way of allocating it between finding (some number of) copies and (partial) manipulation? Formally analyzing this model can put our results into a broader context.

(e) Relationship to other measures. Comparing our results with prior work, there appears to be no clear correlation between the manipulation potential of a rule and how it fares against other notions of manipulability. For example, while we have shown that Borda Count (*BC*) has a significantly lower manipulation potential than other rules (at least with many voters), several papers that study the frequency under which a manipulation can occur (see Section 1.2) conclude that *BC* is "more manipulable" than other rules in this regard [30, 44, 85]. This is not surprising, since the fraction of profiles in which a voter is able to manipulate does not reveal what she could have gained in those profiles by increasing her voter weight instead; thus, it is incomparable with our resource-augmentation approach. On the other hand, there are rules that satisfy weaker forms of strategyproofness, but end up having very large manipulation potentials. For example, various majoritarian Condorcet extensions satisfy "weak-strategyproofness" when assuming voters do not know what tiebreaker is going to be used and are acting safely [18, 23], but all such rules have basically the highest possible manipulation potential for any majority-consistent rule (Theorem 4.14). For other rules that achieve weak-strategyproofness, the manipulation potential can be unbounded (see Section B.12), demonstrating that without the aforementioned assumptions on voters' relationship with ties, no number of truthful copies can match what a voter can achieve by manipulating. A compelling avenue of future research is to combine our metric with other degrees of manipulability. This direction can, for example, entail computing the *expected* manipulation potential of rules under different voter distributions / empirical data, or defining an analogously weaker version k-ASP (Definition 3.1) that incorporates voter uncertainty about the tiebreaker.

B MISSING PROOFS

B.1 Proof of Theorem 4.2

Theorem 4.2. *The manipulation potential of plurality is* $\mathsf{MP}(PL) = \lceil \frac{n-1}{2} \rceil$. *Further, PL is not k-ASP for any* $0 \leq k < \lceil \frac{n-1}{2} \rceil$.

Proof. We first prove that plurality is *not* k-ASP for any $0 \leq k < \lceil \frac{n-1}{2} \rceil$. To do so, we fix any such k and provide an instance (*i.e.*, a specific profile, tiebreaker, and manipulation) where voter 1 strictly prefers the manipulation to adding k copies of her truthful vote. Say the profile $\succ_N$ is:

Voter 1	$c_1 \succ_1 c_2 \succ_1 \ldots \succ_1 c_m$
$\lfloor \frac{n-1}{2} \rfloor$ voters	$c_2 \succ_i \ldots$
$\lceil \frac{n-1}{2} \rceil$ voters	$c_3 \succ_i \ldots$

If n is odd, say the tiebreaker ranks $c_3 \succ_T c_2 \succ_T c_1$; otherwise, say $c_2 \succ_T c_3 \succ_T c_1$. If we add k truthful copies of voter 1 (*i.e.*, $\succ_N +k(\succ_1)$), the number of voters that have c_1, c_2, c_3 as their top candidate will be $k+1$, $\lfloor \frac{n-1}{2} \rfloor$, and $\lceil \frac{n-1}{2} \rceil$, respectively. We cannot have $PL_T(\succ_N +k(\succ_1)) = c_1$ since $k+1 \leq \lceil \frac{n-1}{2} \rceil$ and $c_3 \succ_T c_1$, so PL_T will pick c_3 over c_1. Similarly, c_2 cannot win: if n is even, it has strictly fewer top-choice votes than c_3; if n is odd, they are tied and the tiebreaker $\succ_T$ breaks the tie in favor of c_3. We therefore have $PL_T(\succ_N +k(\succ_1)) = c_3$. Now, go back to the original profile $\succ_N$ (no additional copies of voter 1), and say voter 1 reports an alternative preference order $\succ_1'$ that ranks c_2 top. If n is odd, c_2 has $\frac{n+1}{2}$ voters ranking it top while c_3 has $\frac{n-1}{2}$. If n is even, they both have $\frac{n}{2}$ voters ranking them top, and $\succ_T$ breaks the tie in favor of c_2. Hence, $PL_T(\succ_1', \succ_{-1}) = c_2 \succ_1 c_3 = PL_T(\succ_N +k(\succ_1))$, implying plurality is not k-ASP for any $0 \leq k < \lceil \frac{n-1}{2} \rceil$. We thus have $\mathsf{MP}(PL) \geq \lceil \frac{n-1}{2} \rceil$.

For the upper bound, we need to show that plurality is k-ASP for any $k \geq \lceil \frac{n-1}{2} \rceil$. To do so, fix some profile $\succ_N$, tiebreaker $\succ_T$, manipulation $\succ_1'$, and integer $k \geq \lceil \frac{n-1}{2} \rceil$. We will prove that $PL_T(\succ_N +k(\succ_1)) \succeq_1 PL_T(\succ_1', \succ_{-1})$. First, we rule out two simple cases. If $c_1 \in PL(\succ_N)$ (*i.e.*, c_1 is *one of the* plurality winners of $\succ_N$, pre-tiebreaking), then adding k copies of $\succ_1$ will make it the sole plurality winner (since $k \geq 1$), implying $PL_T(\succ_N +k(\succ_1)) = c_1 \succeq_1 PL_T(\succ_1', \succ_{-1})$ since c_1 is the top choice in $\succ_1$. Similarly, if $PL_T(\succ_N) = PL_T(\succ_1', \succ_{-1})$ (*i.e.*, voter 1's manipulation does not change the winner), we will indeed have $PL_T(\succ_N +k(\succ_1)) \succeq_1 PL_T(\succ_1', \succ_{-1})$, since adding more copies of $\succ_1$ to $\succ_N$ can either not change anything or make c_1 the winner after getting sufficiently many votes. The only remaining case is when $c_1 \notin PL(\succ_N)$ and $PL_T(\succ_N) = c \neq c' = PL_T(\succ_1', \succ_{-1})$ for some $c, c' \in C$. The only way voter 1 could have caused this change is by submitting a vote $\succ_1'$ that ranks c' top instead of c_1 (we cannot have $c' = c_1$, since the number of voters ranking c_1 top can only decrease when going from $\succ_N$ to $(\succ_1', \succ_{-1})$, whereas this number can only increase for any other candidate, implying $c_1 \notin PL(\succ_1', \succ_{-1})$ since $c_1 \notin PL(\succ_N)$ by assumption). If n_c and $n_{c'}$ are the number of voters that rank c and c' top in $\succ_N$, respectively, then we must have $n_{c'} \geq n_c - 1$, since otherwise $c' \notin PL(\succ_1', \succ_{-1})$ (as the top choice of votes of c' could have only gone up by one), which contradicts $c' = PL_T(\succ_1', \succ_{-1})$. Since voter 1 ranks $c_1 \notin \{c, c'\}$ top in $\succ_N$, this implies we have $n \geq n_c + n_{c'} + 1 \geq 2n_c \Rightarrow n_c \leq \frac{n}{2}$. Since c_1 has $k+1 \geq \lceil \frac{n-1}{2} \rceil + 1 > \frac{n}{2}$ voters that rank it top in $\succ_N +k(\succ_1)$, this implies $PL_T(\succ_N +k(\succ_1)) = c_1 \succeq_1 PL_T(\succ_1', \succ_{-1})$, completing our proof. □

B.2 Proof of Theorem 4.3

Theorem 4.3. *The manipulation potentials of Plurality with Runoff & Instant Runoff are* $\mathsf{MP}(PLR) = \mathsf{MP}(IRV) = n-1$ *(lower bound assumes* $m \geq 4$, $n \geq 5$ *and that* $n-1$ *is divisible by 4). Further, neither rule is k-ASP for any* $0 \leq k < n-1$.

Proof. The upper bound for both rules follow from Claim 4.1. It remains to prove that neither PLR nor IRV is k-ASP for any $0 \leq k < n-1$. Fix such a k, and consider following profile $\succ_N$:

Voter 1	$c_1 \succ_1 c_2 \succ_1 \ldots \succ_1 c_m$
$\frac{n-1}{4}$ voters	$c_2 \succ_i c_3 \succ_i c_4 \succ_i \ldots$
$\frac{n-1}{4}$ voters	$c_3 \succ_i c_4 \succ_i c_2 \succ_i \ldots$
$\frac{n-1}{2}$ voters	$c_4 \succ_i \ldots$

Say the tiebreaker $\succ_T$ ranks c_4 top. We first show that $PLR_T(\succ_N +k(\succ_1)) = IRV_T(\succ_N +k(\succ_1)) = c_4$. To do so, it suffices to prove c_4 wins under *some* way of breaking ties in both rules—*i.e.*, $c_4 \in PLR(\succ_N +k(\succ_1)) \cap IRV(\succ_N +k(\succ_1))$—as $\succ_T$ will break the tie in favor of c_4. Consider two cases.

Case 1: $0 \leq k \leq \frac{n-5}{4}$: Here, c_4 starts with strictly more voters ranking it top than any other candidate; c_2 and c_3 tie for second place; and c_1 has weakly less points than any other candidate. With Plurality with Runoff (PLR), if c_2 is advanced to the runoff with c_4, then c_4 wins. With Instant Runoff (IRV), if c_1 is eliminated first, it will be followed by c_3 and then c_4, once again making c_4 win. Hence, c_4 is among the winners for both rules, and therefore is the final winner picked by $\succ_T$.

Case 2: $\frac{n-5}{4} < k < n-1$: Here, c_2 and c_3 tie for least number of top-choice votes. With PLR, c_1 is advanced to the runoff with c_4, but then loses to c_4 since it has $k+1 \leq n-1$ points in the second round. With IRV, if c_3 is eliminated first, it will be followed by c_2 and then c_1, once again making c_4 the winner. Hence, c_4 is among the winners for both rules and thus the final winner picked by $\succ_T$.

Next, go back to the original profile (with no copies) and consider the case where voter 1 reports an alternative preference order $\succ_1'$ ranking c_3 as her top candidate. For Plurality with Runoff (PLR), c_3 and c_4 are advanced to the runoff, and in the second round c_3 wins. For Instant Runoff (IRV), c_1 is eliminated first, followed by c_2 and then c_4, once again making c_3 the winner. Hence, $PLR_T(\succ_1', \succ_{-1}) = IRV_T(\succ_1'$

$, \succ_{-1}) = c_3 \succ_1 c_4 = PLR_T(\succ_N + k(\succ_1)) = IRV_T(\succ_N + k(\succ_1))$. This proves that neither rule is k-ASP for any $0 \le k < n-1$, completing the proof. □

B.3 Proof of Theorem 4.4

Theorem 4.4. *The manipulation potential of Borda Count is* $\mathtt{MP}(BC) = m-2$ *(lower bound assumes n is even). Further, BC is not k-ASP for any* $0 \le k < m-2$.

Proof. We first prove that Borda Count is not k-ASP for any $0 \le k < m-2$, which also implies $\mathtt{MP}(BC) \ge m-2$. Fix any such k and separate the candidates into two groups: $\{c_1, c_2\}$ and $\{c_3, c_4, \ldots, c_m\}$ (colored differently for presentation purposes). Consider a profile $\succ_N$, consisting of:

Voter 1	$c_1 \succ_1 c_2 \succ_1 c_3 \succ_1 c_4 \succ_1 \ldots \succ_1 c_m$
Voter 2	$c_2 \succ_2 c_m \succ_2 c_{m-1} \succ_2 \ldots \succ_2 c_3 \succ_2 c_1$
$\frac{n-2}{2}$ voters (Group 1)	$c_2 \succ_i c_1 \succ_i c_3 \succ_i c_4 \succ_i \ldots \succ_i c_m$
$\frac{n-2}{2}$ voters (Group 2)	$c_1 \succ_i c_2 \succ_i c_m \succ_i c_{m-1} \succ_i \ldots \succ_i c_3$

Assume that the tiebreaker is identical to voter 1's preference order, *i.e.*, $\succ_T = \succ_1$. For each candidate $c \in C$, let $S_k(c)$ denote its total Borda score from all voters in $\succ_N + k(\succ_1)$. Then, we have

$$S_k(c_1) = \underbrace{(k+1)(m-1)}_{\text{Voter 1 and her copies}} + \underbrace{0}_{\text{Voter 2}} + \underbrace{\frac{n-2}{2} \cdot (m-2)}_{\text{Group 1}} + \underbrace{\frac{n-2}{2} \cdot (m-1)}_{\text{Group 2}},$$

$$S_k(c_2) = \underbrace{(k+1)(m-2)}_{\text{Voter 1 and her copies}} + \underbrace{(m-1)}_{\text{Voter 2}} + \underbrace{\frac{n-2}{2} \cdot (m-1)}_{\text{Group 1}} + \underbrace{\frac{n-2}{2} \cdot (m-2)}_{\text{Group 2}}, \text{ and}$$

$$\forall \ell \in \{3, \ldots, m\}: \quad S_k(c_\ell) = \underbrace{(k+1)(m-\ell)}_{\text{Voter 1 and her copies}} + \underbrace{(\ell-2)}_{\text{Voter 2}} + \underbrace{\frac{n-2}{2} \cdot (m-\ell)}_{\text{Group 1}} + \underbrace{\frac{n-2}{2} \cdot (\ell-3)}_{\text{Group 2}}.$$

As $S_k(c_2) - S_k(c_1) = (m-1) - (k+1) > 0$ and $S_k(c_2) - S_k(c_\ell) = k(\ell-2) + \frac{nm}{2} - 1 > 0$ for any $\ell \in \{3, \ldots, m\}$, we get that $BC_T(\succ_N + k(\succ_1)) = c_2$. Now consider an alternative preference order $\succ_1'$ defined as $c_1 \succ_1' c_3 \succ_1' c_4 \succ_1' \ldots \succ_1' c_m \succ_1' c_2$ (*i.e.*, voter 1 moves c_2 to the bottom of her ranking). For each $c \in C$, let $S'(c)$ denote its total Borda score in $(\succ_1', \succ_{-1})$. Then $S'(c_1) - S'(c_2) = 0$ and $S'(c_1) - S'(c_\ell) = \frac{(n-2)m}{2} \ge 0$ for any $\ell \in \{3, \ldots, m\}$, so $c_1 \in BC(\succ_N)$. Since $\succ_T$ ranks c_1 top, we have $BC_T(\succ_1', \succ_{-1}) = c_1 \succ_1 c_2 = BC_T(\succ_N + k(\succ_1))$, proving BC is not k-ASP for any $0 \le k < m-2$.

For the upper bound, fix some $k \ge m-2$, profile $\succ_N$, manipulation $\succ_1'$ and tiebreaker $\succ_T$. We will show that $BC_T(\succ_N + k(\succ_1)) \succeq_1 BC_T(\succ_1', \succ_{-1})$, proving BC is k-ASP. Say B, B', B'' are (all) the Borda winners in $\succ_N$, $(\succ_1', \succ_{-1})$, and $\succ_N + k(\succ_1)$, respectively, prior to tiebreaking. For each candidate $c \in C$, let $S(c), S'(c), S''(c)$ be its respective Borda scores in these three profiles, and let $S_1(c), S_1'(c), S_1''(c)$ denote voter 1's contributions to these scores. As all other votes are unchanged, $S'(c) = S(c) - S_1(c) + S_1'(c)$ and $S''(c) = S(c) + kS_1(c)$. Next, we prove that, for any $c' \in B'$ and $c'' \in B''$, either $c'' \succeq_1 c'$, or $\{c', c''\} \subseteq B' \cap B''$. This will complete the proof, as taking $c' = \max_{\succ_T} B'$ and $c'' = \max_{\succ_T} B''$ implies $BC_T(\succ_N + k(\succ_1)) = \max_{\succ_T} B'' \succeq_1 \max_{\succ_T} B' = BC_T(\succ_1', \succ_{-1})$ (either directly or by the consistency of the tie-breaker T). To prove our claim on *any* $c' \in B'$ and $c'' \in B''$, let $c^* := \max_{\succ_1} B$ (*i.e.*, voter 1's top candidate in B), and consider the following two cases:

Case 1: $c^* \succeq_1 c'$. Since $S''(c'') \ge S''(c^*)$ and $S(c^*) \ge S(c'')$, we have $kS_1(c'') = S''(c'') - S(c'') \ge S''(c^*) - S(c^*) = kS_1(c^*)$, implying $S_1(c'') \ge S_1(c^*)$ (since $k \ge m-2 \ge 1$) and therefore $c'' \succeq_1 c^*$. Since $c^* \succeq_1 c'$, this implies $c'' \succeq_1 c'$, and we are done.

Case 2: $c' \succ_1 c^*$ and thus $S_1(c') - S_1(c^*) \ge 1$. Since $S_1'(c') - S_1'(c^*) \le m-1$, this implies

$$m-2 \ge (S_1'(c') - S_1(c')) - (S_1'(c^*) - S_1(c^*)) = (S'(c') - S(c')) - (S'(c^*) - S(c^*)) \tag{1}$$

$$\ge S(c^*) - S(c'), \tag{2}$$

where the last step follows from $S'(c') \ge S'(c^*)$, since $c' \in B'$. Assume $c' \succ_1 c''$ (otherwise we are done); we will show $\{c', c''\} \subseteq B' \cap B''$. Since $S''(c'') \ge S''(c')$ and $S_1(c') \ge S_1(c'') + 1$, we have

$$S(c'') + kS_1(c'') = S''(c'') \ge S''(c') = S(c') + kS_1(c') \ge S(c^*) - (m-2) + k(1 + S_1(c'')) \tag{3}$$

$$\ge S(c^*) + kS_1(c''), \tag{4}$$

implying $S(c'') \ge S(c^*)$ and thus $c'' \in B$. In fact, we must have $c'' = c^*$, since any $c \in B \setminus \{c^*\}$ will have $S''(c) = S(c) + kS_1(c) = S(c^*) + kS_1(c) < S(c^*) + kS_1(c^*) = S''(c^*)$, so c cannot be in B''. This implies (3)-(4) is in fact an equality, and therefore $S''(c'') = S''(c')$ and $S(c') = S(c^*) - (m-2)$. The latter implies (1)-(2) is also an equality, and therefore $S'(c') = S'(c^*)$. Since $S''(c') = S''(c'')$ and $S'(c'') = S'(c^*) = S'(c')$, and since $c' \in B'$ and $c'' \in B''$, this implies $\{c', c''\} \subseteq B' \cap B''$, as desired.

□

B.4 Proof of Theorem 4.6

Theorem 4.6. *For any F^s with $s_1 = s_2$, we have* $\mathsf{MP}(F^s) = \infty$.

Proof. Assume $s_1 = s_2 = 1$. We will give a profile $\succ_N$, manipulation $\succ_1'$, and tiebreaker $\succ_T$ such that $F_T^s(\succ_1', \succ_{-1}) \succ_1 F_T^s(\succ_N + k(\succ_1))$ for any $k > 0$. Consider a profile $\succ_N$ where *every* voter votes $c_1 \succ c_2 \succ \ldots \succ c_m$ and the tiebreaking order ranks $c_2 \succ_T c_1 \succ_T c$ for any $c \in C \setminus \{c_1, c_2\}$. For any $c \in C$, say $S_k(c)$ is the total score assigned by F^s to c is $\succ_N + k(\succ_1)$. Then we have $S_k(c_2) = (k+n)s_2 = k + n \geq S_k(c)$ for any $c \in C \setminus \{c_2\}$. Since $c_2 \succ_T c$ for any $c \in C \setminus \{c_2\}$, this implies $F_T^s(\succ_N + k(\succ_1)) = c_2$ for all $k \geq 0$. Assume voter 1 instead reported a preference order $\succ_1'$ such that $c_1 \succ_1' c \succ_1' c_2$ for any $c \in C \setminus \{c_1, c_2\}$, and say $S'(c)$ is the total score assigned by F^s to any $c \in C$ in $(\succ_1', \succ_{-1})$. We have $S'(c_2) = (n-1)s_2 + s_m = n - 1$ and $S'(c_1) = ns_1 = n \geq S'(c)$ for any $c \in C$. Since $S'(c_1) \geq S'(c)$ and $c_1 \succ_T c$ for any $c \in C \setminus \{c_1, c_2\}$ and since $S'(c_1) > S'(c_2)$, this implies $F_T^s(\succ_1', \succ_{-1}) = c_1 \succ_1 c_2 = F_T^s(\succ_N + k(\succ_1))$ for any $k \geq 0$, completing our proof. □

B.5 Proof of Theorem 4.7

Theorem 4.7. *For any F^s with $s_1 \neq s_2$, we have that* $\mathsf{MP}(F^s) \leq \max_{\ell \in \{1,2,\ldots,m-1\}} U_s(\ell)$, *where*

$$U_s(\ell) := \begin{cases} \frac{1}{s_\ell - s_{\ell+1}} - 1 & \text{if } \ell = 1 \\ \min\left(\frac{1}{s_\ell - s_{\ell+1}} - 1, \frac{(n-1)(\overline{s}(2) - \underline{s}(\alpha_\ell^s)) + \frac{1}{2}}{\overline{s}(\alpha_\ell^s) - s_{\ell+1}}\right) & \text{otherwise} \end{cases}.$$

Proof. Fix some $\succ_N$, manipulation $\succ_1'$, tiebreaker $\succ_T$, and some $k \geq \max_{\ell \in \{1,2,\ldots,m-1\}} U_s(\ell)$. We will prove that $F_T^s(\succ_N + k(\succ_1)) \succeq_1 F_T^s(\succ_1', \succ_{-1})$. As usual, the truthful preference order of voter 1 is $c_1 \succ_1 c_2 \succ_1 \ldots \succ_1 c_m$. For any candidate $c \in C$, say $S(c), S'(c)$, and $S''(c)$ are the total scores assigned to c by F^s in $\succ_N$, $(\succ_1', \succ_{-1})$, and $\succ_N + k(\succ_1)$, respectively. Say $S_1(c), S_1'(c)$, and $S_1''(c)$ are the contributions of voter 1 to $S(c), S'(c)$ and $S''(c)$, respectively. Lastly, say $S_{-1}(c) = S(c) - S_1(c), S_{-1}'(c) = S'(c) - S_1'(c)$, and $S_{-1}''(c) = S''(c) - S_1''(c)$. First notice that since $k \geq 0$, we have $F_T^s(\succ_N + k(\succ_1)) \succeq_1 F_T^s(\succ_N)$, since adding more copies of voter 1 can only help the candidates it prefers to $F_T^s(\succ_N)$. So if $F_T^s(\succ_N) \succeq_1 F_T^s(\succ_1', \succ_{-1})$, we are done. Otherwise, assume we have $F_T^s(\succ_1', \succ_{-1}) = c_\ell \succ_1 c_i = F_T^s(\succ_N)$ for some $\ell < i$. Since $c_\ell \in B'$, we have $S'(c_\ell) \geq S'(c_i)$, with the inequality strict if $c_i \succ_T c_\ell$. As $S_1(c_\ell) = s_\ell$, $S_1(c_i) = s_i$ and $S_1'(c_\ell) - S_1'(c_i) \leq 1$, we have

$$S(c_\ell) = S'(c_\ell) - S_1'(c_\ell) + S_1(c_\ell) \geq S'(c_i) - S_1'(c_i) - 1 + s_\ell = S(c_i) - s_i - 1 + s_\ell, \tag{5}$$

with the inequality strict if $c_i \succ_T c_\ell$. Take any $j \in \{\ell+1, \ell+2, \ldots, m\}$. We will show that $F_T^s(\succ_N + k(\succ_1)) \neq c_j$, hence proving $F_T^s(\succ_N + k(\succ_1)) \succeq_1 c_\ell = F_T^s(\succ_1', \succ_{-1})$ as desired. Since $F_T^s(\succ_N) = c_i$ we have either

$$S(c_i) > S(c_j) \text{ or } [S(c_i) = S(c_j) \text{ and } c_i \succeq_T c_j]. \tag{6}$$

To complete the proof, we will consider two cases.

Case 1: $U_s(\ell) = \frac{1}{s_\ell - s_{\ell+1}} - 1$, so $k \geq \frac{1}{s_\ell - s_{\ell+1}} - 1$. In this case, we must have $s_\ell > s_{\ell+1}$. Combining (5) and (6), we get

$$\begin{aligned} S''(c_\ell) = S(c_\ell) + ks_\ell &\geq S(c_i) + s_\ell - s_i - 1 + ks_\ell \geq S(c_j) + s_\ell - s_i - 1 + k(s_\ell - s_j) + ks_j \\ &\geq S''(c_j) + s_\ell - s_{\ell+1} - 1 + k(s_\ell - s_{\ell+1}) \geq S''(c_j), \end{aligned}$$

with the inequality strict if $c_i \succ_T c_\ell$ or $c_j \succ_T c_i$. This implies we have either $S''(c_\ell) > S''(c_j)$ or $[S''(c_\ell) = S''(c_j)$ and $c_\ell \succ_T c_i \succeq_T c_j]$ (recall that $\ell \neq i$). Either case implies $F_T^s(\succ_N + k(\succ_1)) \neq c_j$, completing the proof.

Case 2: $U_s(\ell) = \frac{(n-1)(\overline{s}(2) - \underline{s}(\alpha_\ell^s)) + \frac{1}{2}}{\overline{s}(\alpha_\ell^s) - s_{\ell+1}}$, so $k \geq \frac{(n-1)(\overline{s}(2) - \underline{s}(\alpha_\ell^s)) + \frac{1}{2}}{\overline{s}(\alpha_\ell^s) - s_{\ell+1}}$. We will show that there exists a $t \in \{1, 2, \ldots, \alpha_\ell^s\}$ such that $S''(c_t) > S''(c_j)$, which completes the proof. Since each voter can rank at most one candidate in each position, we have

$$\frac{1}{\alpha_\ell^s} \sum_{t=1}^{\alpha_\ell^s} S_{-1}(c_t) \geq \frac{(n-1)}{\alpha_\ell^s} \sum_{t'=m-\alpha_\ell^s+1}^{m} s_{t'} = (n-1) \cdot \underline{s}(\alpha_\ell^s), \tag{7}$$

since any α_ℓ^s candidates must get (in total) at least the bottom α_ℓ^s scores from each voter. Similarly, we have $S_{-1}(c_j) + S_{-1}(c_\ell) \leq (n-1)(s_1 + s_2) = 2(n-1) \cdot \overline{s}(2)$, since any two candidates can get (in total) at most the top two scores from each voter. Combining this with (5) and (6), we get

$$\begin{aligned} 2S_{-1}(c_j) - 1 = S_{-1}(c_j) + S(c_j) - s_j - 1 &\leq S_{-1}(c_j) + (S(c_\ell) + s_i + 1 - s_\ell) - s_j - 1 \\ &= S_{-1}(c_j) + S_{-1}(c_\ell) + s_i - s_j \\ &\leq 2(n-1) \cdot \overline{s}(2) + s_i - s_j \\ \Rightarrow S_{-1}(c_j) &\leq (n-1) \cdot \overline{s}(2) + \frac{1 + s_i - s_j}{2}. \end{aligned}$$

Together with (7), this implies

$$\begin{aligned}
S''(c_j) - \frac{1}{\alpha_\ell^s}\sum_{t=1}^{\alpha_\ell^s} S''(c_t) &= \left(S_{-1}(c_j) + (k+1)s_j\right) - \frac{1}{\alpha_\ell^s}\sum_{t=1}^{\alpha_\ell^s}\left(S_{-1}(c_t) + (k+1)s_t\right) \\
&\le (n-1)\cdot\bar{s}(2) + \frac{1+s_i-s_j}{2} + (k+1)s_j - (n-1)\cdot\underline{s}(\alpha_\ell^s) - (k+1)\cdot\bar{s}(\alpha_\ell^s) \\
&= (n-1)\cdot(\bar{s}(2) - \underline{s}(\alpha_\ell^s)) + \frac{1+s_i-s_j}{2} - (k+1)\cdot(\bar{s}(\alpha_\ell^s) - s_j) \\
&= (n-1)\cdot(\bar{s}(2) - \underline{s}(\alpha_\ell^s)) + \frac{1+s_i+s_j}{2} - \bar{s}(\alpha_\ell^s) - k\cdot(\bar{s}(\alpha_\ell^s) - s_j) \\
&\le (n-1)\cdot(\bar{s}(2) - \underline{s}(\alpha_\ell^s)) + \frac{1}{2} + s_{\ell+1} - \bar{s}(\alpha_\ell^s) - k\cdot(\bar{s}(\alpha_\ell^s) - s_{\ell+1}) \\
&< (n-1)\cdot(\bar{s}(2) - \underline{s}(\alpha_\ell^s)) + \frac{1}{2} - k\cdot(\bar{s}(\alpha_\ell^s) - s_{\ell+1}) \\
&\le (n-1)\cdot(\bar{s}(2) - \underline{s}(\alpha_\ell^s)) + \frac{1}{2} - \frac{(n-1)(\bar{s}(2) - \underline{s}(\alpha_\ell^s)) + \frac{1}{2}}{\bar{s}(\alpha_\ell^s) - s_{\ell+1}}\cdot(\bar{s}(\alpha_\ell^s) - s_{\ell+1}) \\
&= 0.
\end{aligned}$$

By an averaging argument, this implies that there exists some $t \in \{1, \dots \alpha_\ell^s\}$ such that $S''(c_t) > S''(c_j)$. Hence, $F_T^s(\succ_N + k(\succ_1)) \neq c_j$, completing the proof. □

B.6 Proof of Proposition 4.9

Proposition 4.9. *For any F^s with $s_1 \neq s_2$, we have $\mathtt{MP}(F^s) \geq \max_{\ell \in \{1,2,\dots,m-1\}} L_s(\ell)$, where*

$$L_s(\ell) := \min\left(\frac{1}{s_\ell - s_{\ell+1}} - 1,\ \min_{t \in \{1,2,\dots,\alpha_\ell^s\}} \left\lfloor \frac{(n-2)(\bar{s}(2) - s_{m-t+1}) + 1 - s_{m-t}}{s_t - s_{\ell+1}} \right\rfloor\right),$$

assuming n is even and at least $\frac{2s_2}{1-s_2}$. Further, F^s is not k-ASP for any $k < \max_{\ell \in \{1,2,\dots,m-1\}} L_s(\ell)$.

Proof. Fix some $\ell \in \{1, \dots, m-1\}$ and $0 \leq k < L_s(\ell)$ and assume $n \geq \frac{4s_3 - 2s_2}{1-s_2}$. Separate the candidates into three three groups $\{c_1, c_2, \dots, c_{\ell-1}\}$, $\{c_\ell, c_{\ell+1}\}$, and $\{c_{\ell+2}, c_{\ell+3}, \dots, c_m\}$; note that the first (resp. last) group will be empty if $\ell = 1$ (resp. $\ell = m-1$). For presentation purposes, we will use a different color while referring to candidates from each group. Consider the following profile, consisting of

- voter 1: $c_1 \succ_1 \dots \succ_1 c_{\ell-1} \succ_1 c_\ell \succ_1 c_{\ell+1} \succ_1 c_{\ell+2} \succ_1 \dots \succ_1 c_m$,
- voter 2: $c_{\ell+1} \succ_2 c_m \succ_2 c_{m-1} \succ_2 \dots \succ_2 c_{\ell+2} \succ_2 c_{\ell-1} \succ_2 c_{\ell-2} \succ_2 \dots \succ_2 c_1 \succ_2 c_\ell$,
- $\frac{n-2}{2}$ voters (Group 1): $c_\ell \succ c_{\ell+1} \succ c_m \succ c_{m-1} \succ_2 \dots \succ c_{\ell+2} \succ c_{\ell-1} \succ c_{\ell-2} \succ \dots \succ c_1$, and
- $\frac{n-2}{2}$ voters (Group 2): $c_{\ell+1} \succ c_\ell \succ c_m \succ c_{m-1} \succ_2 \dots \succ c_{\ell+2} \succ c_{\ell-1} \succ c_{\ell-2} \succ \dots \succ c_1$.

Say the tiebreaker ranks $c_\ell \succ_T c_{\ell+1} \succ_T c$ for all $c \in C \setminus \{c_\ell, c_{\ell+1}\}$. For any $c \in C$, say $S_k(c)$ is the total score assigned to c by F^s on $\succ_N + k(\succ_1)$. For any $t > \ell + 1$, we have $S_k(c_{\ell+1}) \geq S_k(c_t)$, since $c_{\ell+1}$ is ranked above c_t by all voters. Further, since $k < L_s(\ell)$, we have

$$\begin{aligned}
S_k(c_\ell) - S_k(c_{\ell+1}) &= (k+1)(s_\ell - s_{\ell+1}) - 1 < 0, \text{ and} \\
\forall t < \ell : S_k(c_t) - S_k(c_{\ell+1}) &= (k+1)(s_t - s_{\ell+1}) + s_{m-t} - 1 + (n-2)(s_{m-t+1} - \bar{s}(2)) \leq 0.
\end{aligned}$$

Thus, for all $c \in C \setminus \{c_{\ell+1}\}$, we have either $S_k(c_{\ell+1}) > S_k(c)$ or $[S_k(c_{\ell+1}) \geq S_k(c)$ and $c_{\ell+1} \succ_T c]$, implying that $F_T^s(\succ_N + k(\succ_1)) = c_{\ell+1}$.

Now assume voter 1 instead reports the preference order $c_\ell \succ'_1 c_m \succ'_1 c_{m-1} \succ'_1 \dots \succ'_1 c_{\ell+2} \succ'_1 c_{\ell-1} \succ'_1 c_{\ell-2} \succ'_1 \dots \succ'_1 c_1 \succ'_1 c_{\ell+1}$. For each $c \in C$, say $S'(c)$ is the total score assigned to c by F^s on $(\succ'_1, \succ_{-1})$. Then we have $S'(c_\ell) = S'(c_{\ell+1}) = 1 + (n-2)\cdot\bar{s}(2)$, whereas for any other candidate $c \in C \setminus \{c_\ell, c_{\ell+1}\}$, we have

$$\begin{aligned}
S'(c) &\leq 2s_2 + (n-2)s_3 \leq ns_2 \\
\Rightarrow S'(c_\ell) - S'(c) &\geq 1 - 2s_2 + (n-2)\cdot(\bar{s}(2) - s_2) = 1 - 2s_2 + \frac{(n-2)(1-s_2)}{2} \geq 0
\end{aligned}$$

since $n \geq \frac{2s_2}{1-s_2}$. This implies that for all $c \in C \setminus \{c_\ell\}$, we have $S'(c_\ell) \geq S'(c)$ and $c_\ell \succ_T c$, implying $F_T^s(\succ'_1, \succ_{-1}) = c_\ell \succ_1 c_{\ell+1} = F_T^s(\succ_N + k(\succ_1))$. Hence, F^s cannot be k-ASP for any $k < L_s(\ell)$, implying $\mathtt{MP}(F^s) \geq L_s(\ell)$. Since this is true for all $\ell \in \{1, 2, \dots, m-1\}$, we get the theorem statement.

□

B.7 Proof of Theorem 4.10

Theorem 4.10. *For any positional scoring rule $F^s \notin \{PL, BC\}$, given some $n \geq 4$ that is divisible by 4, we have either* $\mathsf{MP}(F^s) > \mathsf{MP}(BC)$ *or* $\mathsf{MP}(F^s) \geq \mathsf{MP}(PL)$*. Further, if $n \geq \max(6, \frac{2}{s_2}+4)$, then either* $\mathsf{MP}(F^s) > \mathsf{MP}(BC)$ *or* $\mathsf{MP}(F^s) > \mathsf{MP}(PL)$ (i.e., *the second inequality is also strict).*

Proof. Fix some positional scoring rule $F^s \notin \{PL, BC\}$. If $s_1 = s_2$, then $\mathsf{MP}(F^s) = \infty$ by Theorem 4.6, and we are done. Otherwise, we will consider two cases based on the value of $\frac{2s_2}{1-s_2}$.

Case 1: $n < \frac{2s_2}{1-s_2}$. Then consider a profile $\succ_N$ where $\frac{n}{4}$ voters (including voter 1) all rank $c_1 \succ c_2 \succ c$ for all $c \in C \setminus \{c_1, c_2\}$, and the remaining $\frac{3n}{4}$ voters and the tiebreaker $\succ_T$ all rank $c_2 \succ c_1 \succ c$ for all $c \in C \setminus \{c_1, c_2\}$. Fix any $0 \leq k \leq \frac{n}{2}$ and for any $c \in C$ say $S_k(c)$ is the total scored assigned by F^s to c on $\succ_N +k(\succ_1)$. Then $S_k(c_2) - S_k(c_1) = \left(\frac{3n}{4} - \left(\frac{n}{4}+k\right)\right)(1-s_2) = \left(\frac{n}{2}-k\right)(1-s_2) \geq 0$. Similarly, $S_k(c_2) \geq S_k(c)$ for all $c \in C \setminus \{c_1, c_2\}$ since all voters rank $c_2 \succ c$. As c_2 is also ranked top by the tiebreaker $\succ_T$, this implies $F^s(\succ_N +k(\succ_1)) = c_2$.

On the other hand, say voter 1 instead submit a preference order $\succ_1'$ that ranks c_1 first and c_2 last, and for any $c \in C$ say $S'(c)$ is the total score assigned to c by F^s on $(\succ_1', \succ_{-1})$. Then $S'(c_1) - S'(c_2) = \left(\frac{n}{4} + \frac{3n}{4}s_2\right) - \left(\left(\frac{n}{4}-1\right)s_2 + \frac{3n}{4}\right) = s_2 - \frac{n}{2}(1-s_2) > 0$. Further, for any $c \in C \setminus \{c_1, c_2\}$, we have $S'(c_1) \geq S'(c)$ and $c_1 \succ_T c$, implying $F^s(\succ_1', \succ_{-1}) = c_1 \succ_1 c_2 = F^s(\succ_N +k(\succ_1))$. Therefore, F^s cannot be k-ASP for any $k \leq \frac{n}{2}$, implying $\mathsf{MP}(F^s) > \frac{n}{2} \geq \mathsf{MP}(PL)$, as desired.

Case 2: $n \geq \frac{2s_2}{1-s_2}$. In this case, we can invoke Proposition 4.9. We cannot have $s_{i+1} \leq s_i - \frac{1}{m-1}$ for all $i \in \{1, ..., m-1\}$; if we did, this would imply $s_i \leq s_1 - \frac{i-1}{m-1} = \frac{m-i}{m-1}$ and $s_i \geq s_m + \frac{m-i}{m-1} = \frac{m-i}{m-1}$ for all i, meaning $F^s = BC$, which is a contradiction. Hence, let $\ell = \min\{i \in \{1, \ldots, m-1\} : s_i - s_{i+1} < \frac{1}{m-1}\}$. If $\ell = 1$, Proposition 4.9 would imply $\mathsf{MP}(F^s) \geq L_s(1) = \frac{1}{s_1 - s_2} - 1 > m-2 = \mathsf{MP}(BC)$, and we are done. Otherwise, say $\ell \geq 2$, and define

$$B_s(\ell) \coloneqq \frac{1}{s_\ell - s_{\ell+1}} - 1, \text{ and}$$
$$\forall t \in \{1, \ldots, \ell-1\} : P_s(\ell, t) \coloneqq \frac{(n-2)(\bar{s}(2) - s_{m-t+1}) + 1 - s_{m-t}}{s_t - s_{\ell+1}}.$$

By assumption, we have $B_s(\ell) > m-2 = \mathsf{MP}(BC)$. To complete the proof, we will prove that $P_s(\ell, t) \geq \mathsf{MP}(PL)$ (and $P_s(\ell, t) \geq \mathsf{MP}(PL)+1$ if $n \geq \frac{2}{s_2}+2$) for each $t \in \{1, \ldots, \ell-1\}$, which is sufficient since $\mathsf{MP}(F^s) \geq L_s(\ell) = \min(B_s(\ell), \min_t \lfloor P_s(\ell, t) \rfloor)$ by Proposition 4.9. By assumption, we have $s_i - s_{i+1} \geq \frac{1}{m-1}$ for any $i < \ell$. Fix any $t \in \{1, \ldots \ell-1\}$, and consider three subcases.

Case 2a: $t = 1$. Then we have

$$\begin{aligned} P_s(\ell, 1) - \mathsf{MP}(PL) \geq P_s(\ell, 1) - \frac{n}{2} &= \frac{(n-2)(s_1 + s_2 - 2s_m) + 2 - 2s_{m-1} - n(s_1 - s_{\ell+1})}{2(s_1 - s_{\ell+1})} \\ &= \frac{(n-2)s_2 - 2s_{m-1} + ns_{\ell+1}}{2(1 - s_{\ell+1})} \geq \frac{(n-4)s_2}{2} \geq 0 \quad \left(\text{and } \geq 1 \text{ if } n \geq \frac{2}{s_2}+4\right). \end{aligned}$$

Case 2b: $t \geq 2$, $\ell \leq m-t$. In this case we have $s_\ell \geq s_{\ell+1} \geq s_{m-t+1}$ and $s_\ell \geq s_{m-t} \geq s_{m-t+1}$. This implies

$$\begin{aligned} P_s(\ell, t) - \mathsf{MP}(PL) \geq P_s(\ell, t) - \frac{n}{2} &= \frac{(n-2)(s_1 + s_2 - 2s_{m-t+1}) + 2 - 2s_{m-t} - n(s_t - s_{\ell+1})}{2(s_t - s_{\ell+1})} \\ &\geq \frac{(n-2)(1 + s_2 - 2s_{\ell+1}) + 2 - 2s_\ell - n(s_2 - s_{\ell+1})}{2(s_2 - s_{\ell+1})} = \frac{n - 2s_2 - (n-4)s_{\ell+1} - 2s_\ell}{2(s_2 - s_{\ell+1})} \\ &\geq \frac{n - 2s_2 - 2s_\ell}{2s_2} \geq \frac{n(m-1) - 2(m-\ell)}{2(m-2)} - 1 \geq \frac{4(m-1) - 2(m-3)}{2(m-2)} - 1 \geq 0, \end{aligned}$$

since $s_\ell \leq s_1 - \frac{\ell-1}{m-1} = \frac{m-\ell}{m-1}$, $s_2 \leq s_1 - \frac{1}{m-1} = \frac{m-2}{m-1}$, $n \geq 4$, and $\ell \geq 3$ (as $t \geq 2$). Further, if $n \geq 6$, we get a lower bound of 1 instead.

Case 2c: $t \geq 2$, $\ell \geq m-t+1$. Since $s_i \leq \frac{m-i}{m-1}$ for all $i \leq \ell$ and since $s_2 - s_{m-t+1} \geq \frac{m-t-1}{m-1}$, we get

$$\begin{aligned} P_s(\ell, t) - \mathsf{MP}(PL) \geq P_s(\ell, t) - \frac{n}{2} &= \frac{(n-2)(s_1 + s_2 - 2s_{m-t+1}) + 2 - 2s_{m-t} - n(s_t - s_{\ell+1})}{2(s_t - s_{\ell+1})} \\ &\geq \frac{n + (n-2)(s_2 - 2s_{m-t+1}) - 2s_{m-t} - ns_t}{2s_t} \\ &\geq \frac{n(t-1) + (n-2)((m-t-1) - (t-2)) - 2(t-1)}{2(m-t)} = \frac{(n-2)(m-t)}{2(m-t)} \geq 1. \end{aligned}$$

Hence, we have shown that $P_s(\ell, t) \geq \mathsf{MP}(PL)$ (and $P_s(\ell, t) \geq \mathsf{MP}(PL)+1$ whenever $n \geq \max(6, \frac{2}{s_2}+4)$) for all $t \in \{1, \ldots, \ell-1\}$, completing the proof of the theorem. □

B.8 Proof of Theorem 4.11

Theorem 4.11. *The manipulation potential of Black's Rule is* $\mathsf{MP}(BL) = n-1$ *(lower bound assumes $m \geq 4$ and odd n). Further, BL is not k-ASP for any $0 \leq k < n-1$.*

Proof. Upper bound follows from Claim 4.1. For the lower bound, fix any $0 \le k < n-1$. We will show that BL is not k-ASP. Consider the following profile:

Voter 1	$c_1 \succ_1 c_2 \succ_1 c_3 \succ_1 \dots$
$\frac{n-1}{2}$ voters (Group 2)	$c_2 \succ_i c_3 \succ_i \dots \succ_i c_1$
$\frac{n-1}{2}$ voters (Group 3)	$c_3 \succ_i \dots \succ_i c_1 \succ_i c_2$

Say the tiebreaker $\succ_T$ ranks c_3 top. Then we have

$$M(\succ_N + k(\succ_1))[c_1, c_2] = M(\succ_N + k(\succ_1))[c_2, c_3] = k+1 > 0, \text{ and}$$
$$M(\succ_N + k(\succ_1))[c_3, c_1] = (n-1) - (k+1) \ge 0,$$

and any other $c \in C \setminus \{c_1, c_2, c_3\}$ has $M(\succ_N + k(\succ_1))[c_3, c] = n + k$. Hence, there is no Condorcet winner in $\succ_N + k(\succ_1)$, as any candidate has at least one other candidate it does not defeat. For each candidate $c \in C$, say $S_k(c)$ is the total Borda score of c in $\succ_N + k(\succ_1)$. Then we have

$$S_k(c_3) - S_k(c_1) = (n-1)(m-2) - 2(k+1) \ge 0,$$
$$S_k(c_3) - S_k(c_2) = \frac{n-1}{2}(m-2) - (k+1) \ge 0,$$

and $S_k(c_3) - S_k(c) > 0$ for any other c. Since $\succ_T$ ranks c_3 top, this implies $BL(\succ_N + k(\succ_1)) = BC(\succ_N + k(\succ_1)) = c_3$. On the other hand, if Voter 1 reports an alternative preference order $\succ_1'$ ranking c_2 top, she will make it the Condorcet winner in $(\succ_1', \succ_{-1})$. Hence, we have $BL(\succ_1', \succ_{-1}) = c_2 \succ_1 c_3 = BL(\succ_N + k(\succ_1))$, completing the proof. □

B.9 Proof of Theorem 4.12

Theorem 4.12. *The manipulation potential of Maximin is at most $\frac{(m-2)n}{m-1} + 2$ and (assuming $n-2$ is divisible by m) at least $\frac{(m-2)(n-2)}{m-1} + 1$. Further, MM is not k-ASP for any $k < \frac{(m-2)(n-2)}{m-1} + 1$.*

For presentation purposes, will separate the theorem into two propositions (one for the upper bound and one for the lower bound) and prove them separately.

Proposition B.1. *The manipulation potential Maximin is at most $\frac{(m-2)n}{m-1} + 2$.*

Proof. Fix any profile $\succ_N$, tiebreaker $\succ_T$, manipulation $\succ_1'$, and integer $k \ge \frac{(m-2)n}{m-1} + 2$. For each $c, c' \in C$, denote $n_{c \succ c'} := |\{i \in N : c \succ_i c'\}|$ and $d_c := \min_{c' \in C} n_{c \succ c'}$. Thus the maximin winners are $MM(\succ_N) = \arg\max_{c \in C} d_c$, since $M(\succ_N)[c, c'] = 2n_{c \succ c'} - n$ for any $c, c' \in C$. Define $n^*_{c \succ c'}$ and d^*_c analogously for the profile $\succ_N + k(\succ_1)$. Say $MM_T(\succ_N) = c_x$ and $MM_T(\succ_1', \succ_{-1}) = c_y$. We must have

$$d_{c_y} \ge d_{c_x} - 2, \tag{8}$$

since voter 1 can add at most one to d_{c_y} and subtract at most one from d_{c_x} by changing her vote. Construct a simple cycle $(c_{(1)}, c_{(2)}, \dots, c_{(\ell)}, c_{(1)})$ such that $c_{(i+1)} \in \text{argmin}_{c \in C} n_{c_{(i)} \succ c}$ for each $i \in [\ell]$ (where we interpret $\ell + 1 = 1$) and so $d_{c_{(i)}} = n_{c_{(i)} \succ c_{(i+1)}}$ for each $i \in [\ell]$. Note that such a cycle must exist, since each $c \in C$ has a nonempty $\text{argmin}_{c' \in C} n_{c \succ c'}$, and there are a finite number of candidates. Each voter must rank $c_{(i)} \succ c_{(i+1)}$ for at least one $i \in [\ell]$, since voter preferences are acyclic (so disagreeing with every edge in the cycle leads to a contradiction). This implies

$$\sum_{i=1}^{\ell} d_{c_{(i)}} = \sum_{i=1}^{\ell} n_{c_{(i)} \succ c_{(i+1)}} \ge n. \tag{9}$$

Lastly, label the top candidate of voter 1 as $c_z := c_1$ in order (to avoid confusion with $c_{(1)}$). We now prove that MM is k-ASP under two different cases.

Case 1: $\ell \le m-1$. By (9), there exists some $i^* \in [\ell]$ such that $d_{c_{(i^*)}} \ge \frac{n}{\ell} \ge \frac{n}{m-1}$. Since $c_x \in MM(\succ_N)$, we also have $d_{c_x} \ge d_{c_{(i^*)}} \ge \frac{n}{m-1}$. This implies that for any $c \in C$ such that $c_y \succ_1 c$ we have

$$d^*_c \le n^*_{c \succ c_y} = n_{c \succ c_y} = n - n_{c_y \succ c} \le n - d_{c_y} \le n - d_{c_x} + 2 \le \frac{(m-2)n}{m-1} + 2, \tag{10}$$

where the penultimate inequality follows from (8). As $n^*_{c_z \succ c'} = n_{c_z \succ c'} + k$ for any $c' \in C \setminus \{c_z\}$, we have

$$d^*_{c_z} = d_{c_z} + k \ge 1 + k \ge \frac{(m-2)n}{m-1} + 3, \tag{11}$$

where $d_{c_z} \ge 1$ since at least voter 1 ranks c_z above everyone else in $\succ_N$. Comparing (10) with (11), we see that for any $c \in C$ such that $c_y \succ_1 c$, we have $d^*_{c_z} > d^*_c$ and therefore $c \notin MM(\succ_N + k(\succ_1))$. This ensures that $MM_T(\succ_N + k(\succ_1)) \succeq_1 c_y = MM_T(\succ_1', \succ_{-1})$, as desired.

Case 2: $\ell = m$. This implies that all candidates are in the cycle constructed above, implying by (9) that $\sum_{c \in C \setminus \{c_z\}} d_c \ge n - d_{c_z}$. Since $d_{c_x} \ge d_c$ for all $c \in C$, this gives us $d_{c_x} \ge \frac{n - d_{c_z}}{m-1}$. This implies that for any $c \in C$ such that $c_y \succ_1 c$, we have

$$d^*_c \le n^*_{c \succ c_y} = n_{c \succ c_y} = n - n_{c_y \succ c} \le n - d_{c_y} \le n - d_{c_x} + 2 \le \frac{(m-2)n}{m-1} + \frac{d_{c_z}}{m-1} + 2, \tag{12}$$

where the penultimate inequality follows from (8). Since $n^*_{c_z \succ c'} = n_{c_z \succ c'} + k$ for any $c' \in C \setminus \{c_z\}$, we have

$$d^*_{c_z} = d_{c_z} + k \geq d_{c_z} + \frac{(m-2)n}{m-1} + 2 > \frac{d_{c_z}}{m-1} + \frac{(m-2)n}{m-1} + 2,$$

where the last inequality follows from the fact that $d_{c_z} \geq 1$. Comparing this to (12), we see that for any $c \in C$ such that $c_y \succ_1 c$, we have $d^*_{c_z} > d^*_c$ and therefore $c \notin MM(\succ_N + k(\succ_1))$. Once again, this ensures that $MM_T(\succ_N + k(\succ_1)) \succeq_1 c_y = MM_T(\succ'_1, \succ_{-1})$, completing the proof. ◻

Proposition B.2. *The manipulation potential Maximin (MM) is at least $\frac{(m-2)(n-2)}{m-1} + 1$, assuming $n-2$ is divisible by m. Further, MM is not k-ASP for any $k < \frac{(m-2)(n-2)}{m-1} + 1$.*

Proof. Fix some $0 \leq k < \frac{(m-2)(n-2)}{m-1} + 1$. We will show that MM is not k-ASP. To do so, separate the candidates into three groups $\{c_1, c_2, \ldots, c_{m-2}\}$, $\{c_{m-1}\}$, and $\{c_m\}$ and consider the following profile (note that voter 1 has a different ranking than usual, for presentation purposes):

Voter 1	$c_m \succ_1 c_1 \succ_1 c_{m-1} \succ_1 c_2 \succ_1 c_3 \succ_1 \ldots \succ_1 c_{m-2}$
Voter 2	$c_{m-2} \succ_2 c_{m-3} \succ_2 \ldots \succ_2 c_2 \succ_2 c_{m-1} \succ_2 c_1 \succ_2 c_m$
$\frac{n-2}{m-1}$ voters	$c_1 \succ_i c_2 \succ_i \ldots \succ_i c_{m-2} \succ_i c_{m-1} \succ_i c_m$
$\frac{n-2}{m-1}$ voters	$c_{m-1} \succ_i c_1 \succ_i c_2 \succ_i \ldots \succ_i c_{m-2} \succ_i c_m$
$\frac{n-2}{m-1}$ voters	$c_{m-2} \succ_i c_{m-1} \succ_i c_1 \succ_i c_2 \succ_i \ldots \succ_i c_{m-3} \succ_i c_m$
$\frac{n-2}{m-1}$ voters	$c_{m-3} \succ_i c_{m-2} \succ_i c_{m-1} \succ_i c_1 \succ_i c_2 \succ_i \ldots \succ_i c_{m-4} \succ_i c_m$
$\vdots$	$\vdots$
$\frac{n-2}{m-1}$ voters	$c_3 \succ_i c_4 \succ_i \ldots \succ_i c_{m-2} \succ_i c_{m-1} \succ_i c_1 \succ_i c_2 \succ_i c_m$
$\frac{n-2}{m-1}$ voters	$c_2 \succ_i c_3 \succ_i c_4 \succ_i \ldots \succ_i c_{m-2} \succ_i c_{m-1} \succ_i \succ_i c_m \succ_i c_1$

Say that the tiebreaker $\succ_T$ ranks c_{m-1} top and c_1 second. Write $M_k = M(\succ_N + k(\succ_1))$ for convenience. It can then be checked that

- $\forall i \in \{2, 3, \ldots, m-1\} : M_k[c_i, c_m] = n - 2 - k$
- $M_k[c_1, c_m] = \frac{m-3}{m-1}(n-2) - k,$
- $\forall 1 \leq i < j \leq m-2 : M_k[c_i, c_j] = \frac{(m-1)-2(j-i)}{m-1}(n-2) + k,$
- $\forall 2 \leq i \leq m-2 : M_k[c_i, c_{m-1}] = \frac{(m-1)-2(m-1-i)}{m-1}(n-2) - k$, and
- $M_k[c_1, c_{m-1}] = \frac{(m-1)-2(m-2)}{m-1}(n-2) + k.$

Accordingly, we have

- $b_k[c_1] \coloneqq \min_{c \in C \setminus \{c_1\}} M_k[c_1, c] = \min\left(\frac{m-3}{m-1}(n-2) - k,\ k - \frac{m-3}{m-1}(n-2)\right),$
- $\forall i \in \{2, 3, \ldots, m-2\} : b_k[c_i] \coloneqq \min_{c \in C \setminus \{c_i\}} M_k[c_i, c] = -\frac{m-3}{m-1}(n-2) - k,$
- $b_k[c_{m-1}] \coloneqq \min_{c \in C \setminus \{c_{m-1}\}} M_k[c_{m-1}, c] = \min\left(\frac{m-3}{m-1}(n-2) - k,\ k - \frac{m-3}{m-1}(n-2)\right)$, and
- $b_k[c_m] \coloneqq \min_{c \in C \setminus \{c_{m-1}\}} M_k[c_{m-1}, c] = k - (n-2).$

It is easy to check that $b_k[c_{m-1}] \geq b_k[c_i]$ for any $1 \leq i \leq m-2$. Further,

$$b_k[c_{m-1}] - b_k[c_m] = \min\left(\frac{2m-4}{m-1}(n-2) - 2k,\ \frac{2(n-2)}{m-1}\right) \geq \min\left(0,\ \frac{2(n-2)}{m-1}\right) \geq 0.$$

Since $\succ_T$ ranks c_{m-1} first, this implies that $MM_T(\succ_N + k(\succ_1)) = c_{m-1}$. Now, go back to the original profile and say voter 1 reports an alternative preference order $\succ'_1$ that ranks $c_m \succ_1 c_1 \succ_1 c_2 \succ_1 c_3 \succ_1 \ldots \succ_1 c_{m-2} \succ_1 c_{m-1}$. Now, we have:

$$\begin{aligned} \min_{c \in C \setminus \{c_{m-1}\}} M(\succ'_1, \succ_{-1})[c_{m-1}, c] \leq{} & M(\succ'_1, \succ_{-1})[c_{m-1}, c_{m-2}] = -\frac{m-3}{m-1}(n-2) - 2, \\ \min_{c \in C \setminus \{c_m\}} M(\succ'_1, \succ_{-1})[c_m, c] &= -(n-2), \text{ and} \\ \forall i \in \{1, 2, \ldots, m-2\} : \min_{c \in C \setminus \{c_i\}} M(\succ'_1, \succ_{-1})[c_i, c] &= -(n-2), \end{aligned}$$

implying $MM(\succ'_1, \succ_{-1}) = C \setminus \{c_{m-1}, c_m\}$. Since c_1 is ranked second by $\succ_T$ after c_{m-1}, this implies $MM_T(\succ'_1, \succ_{-1}) = c_1 \succ_1 c_{m-1} = MM_T(\succ_N + k(\succ_1))$, completing the proof. ◻

B.10 Proof of Theorem 4.14

Theorem 4.14. *Any SCC F that is (1) neutral (2) biranking-majority-consistent and (3) majoritarian will have* $\mathsf{MP}(F) \geq n-2$. *Further, F is not k-ASP for any $k < n-2$.*

Proof. Fix any $1 \leq k < n-2$ and an F satisfying the conditions above. We will show that F is not k-ASP (The claim for $k = 0$ follows from the Gibbard–Satterthwaite theorem, as any Condorcet extension equipped with a tiebreaker is onto and nondictatorial). Consider the following profile:

Voter 1	$c_1 \succ_1 c_2 \succ_1 c_3 \succ_1 c_4 \succ_1 \ldots \succ_1 c_m$
$\lceil (n-1)/2 \rceil$ voters (Group 1)	$c_2 \succ_i c_3 \succ_i c_1 \succ_i c_4 \succ_i \ldots \succ_i c_m$
$\lfloor (n-1)/2 \rfloor$ voters (Group 2)	$c_3 \succ_i c_1 \succ_i c_2 \succ_2 c_4 \succ_i \ldots \succ_i c_m$

Assume that the tiebreaker ranks $c_3 \succ_T c_2 \succ_T c_1 \succ_T c_4 \succ_T \ldots \succ_T c_m$. Then we have

$$M(\succ_N + k(\succ_1))[c_1, c_2] = (k+1) + \lfloor (n-1)/2 \rfloor - \lceil (n-1)/2 \rceil \geq 1,$$
$$M(\succ_N + k(\succ_1))[c_2, c_3] = (k+1) + \lceil (n-1)/2 \rceil - \lfloor (n-1)/2 \rfloor \geq 2, \text{ and}$$
$$M(\succ_N + k(\succ_1))[c_3, c_1] = \lceil (n-1)/2 \rceil + \lfloor (n-1)/2 \rfloor - (k+1) \geq 1.$$

Thus, c_1, c_2, and c_3 form a 3-cycle in terms of majority defeats (*i.e.*, a Condorcet cycle), and each of them defeats c_ℓ for any $\ell \geq 4$. By neutrality, this implies that $\{c_1, c_2, c_3\} \cap F(\succ_N + k(\succ_1)) \neq \emptyset$ if and only if $\{c_1, c_2, c_3\} \subseteq F(\succ_N + k(\succ_1))$, as we can permute these three candidates in a cycle without changing the graph of majority defeats. As $\succ_T$ ranks c_3 first, this implies that we have $F_T(\succ_N + k(\succ_1)) \notin \{c_1, c_2\}$ and so $c_2 \succ_1 F_T(\succ_N + k(\succ_1))$. Consider a manipulation where voter 1 also reports $c_2 \succ'_1 c_3 \succ'_1 c_1 \succ'_1 c_4 \succ'_1 \ldots \succ'_1 c_m$, joining Group 1. Then, by biranking-majority-consistency, we have $F_T(\succ'_1, \succ_{-1}) = c_2 \succ_1 F_T(\succ_N + k(\succ_1))$, completing the proof. □

B.11 Proof of Theorem 4.15

Theorem 4.15. *Whenever n is odd, any biranking-majority-consistent SCF f has* $\mathsf{MP}(f) \geq \frac{n-1}{2}$.

Proof. Fix some odd n, and consider a profile with $\frac{3(n-1)}{2}$ voters divided into three groups:

Profile 1	
$\frac{n-1}{2}$ voters (Group 1)	$c_1 \succ_i c_2 \succ_i c_3 \succ_i c_4 \succ_i \ldots \succ_i c_m$
$\frac{n-1}{2}$ voters (Group 2)	$c_2 \succ_i c_3 \succ_i c_1 \succ_i c_4 \succ_i \ldots \succ_i c_m$
$\frac{n-1}{2}$ voters (Group 3)	$c_3 \succ_i c_1 \succ_i c_2 \succ_2 c_4 \succ_i \ldots \succ_i c_m$

Since f will output only one winner, at least one out the three groups of voters will get neither their first nor second choice. WLOG, say this is Group 1, *i.e.*, the winner output by f on Profile 1 is not in $\{c_1, c_2\}$. Then remove $k = \frac{n-1}{2} - 1$ copies of Group 1, in order to get a second profile:

Profile 2	
Voter 1	$c_1 \succ_1 c_2 \succ_1 c_3 \succ_1 c_4 \succ_1 \ldots \succ_i c_m$
$\frac{n-1}{2}$ voters (Group 2)	$c_2 \succ_i c_3 \succ_i c_1 \succ_i c_4 \succ_i \ldots \succ_i c_m$
$\frac{n-1}{2}$ voters (Group 3)	$c_3 \succ_i c_1 \succ_i c_2 \succ_2 c_4 \succ_i \ldots \succ_i c_m$

If we label Profile 1 as $\succ_N$, then Profile 2 is just $\succ_N + k(\succ_1)$. Consider a manipulation in $\succ_N$ (Profile 2) where voter 1 also reports the ranking of Group 2. Then, by biranking-majority-consistency, we have $f(\succ'_1, \succ_{-1}) = c_2 \succ_1 f(\succ_N + k(\succ_1))$, showing that F is not $\left(\frac{n-1}{2} - 1\right)$-ASP. □

B.12 Pareto and Omninomination

In this section, we analyze two simple rules.[11] The Pareto rule, which returns all alternatives that are not Pareto dominated, where a candidate a is Pareto dominated if there exists another candidate b such that $b \succ_i a$ for all voters $i \in N$. And the Omninomination rule, which returns all candidates that are the top choice of some voter. We prove that the manipulation potential of both rules is unbounded.

Proposition B.3. *The manipulation potential of Pareto and Omninomination are both ∞.*

Proof. We prove the claim by constructing an instance in which a voter admits a manipulation that strictly improves the outcome over truthful reporting, while adding any number of voters leaves the outcome unchanged. Hence, these rules are *not* k-ASP for any $k \geq 0$, as manipulation always yields a strictly better outcome.

Consider the following election instance:

Voter 1	$c_1 \succ_1 c_2 \succ_1 c_3 \succ_1 c_4 \succ_1 c_5 \succ_1 \ldots \succ_1 c_m$
Voter 2	$c_4 \succ_2 c_3 \succ_2 c_1 \succ_2 c_2 \succ_2 c_5 \succ_2 \ldots \succ_2 c_m$

[11]See Brandt and Brill [19] for more details on the strategic properties of these rules.

Say the tiebreaker ranks $c_2 \succ_T c_4 \succ_T c_3 \succ_T c_1 \succ_T \ldots$ (the rest of the order does not matter).

First, observe that the election outcomes are as follows. Under Pareto, the only candidates that are not Pareto dominated (and hence belong to the winner set before tie breaking) are c_1, c_3 and c_4 (since candidates $\{c_2, c_5, \ldots c_m\}$ are dominated by candidate c_1). After tie-breaking, the winner is therefore c_4.

Under Omninomination, it is easy to see that both (and only) c_1 and c_4 are the top choice of some voter. The tie between them breaks by T in favor of c_4.

We now consider the perspective of voter 1. The first observation is that, indeed, regardless of how many additional copies of voter 1 are added, the outcome remains unchanged. Next, consider the following manipulation for voter 1: $c_2 \succ_1 c_1 \succ_1 c_3 \succ_1 c_4 \succ_1 c_5 \succ_1 \ldots \succ_1 c_m$. Now, under Pareto, we get a tie among c_1, c_2, c_3 and c_4, which breaks by T in favor of c_2. Under Omninomination, the new outcome is a tie between c_2 and c_4, which again breaks in favor of c_2.

Hence, in both cases the manipulation of voter 1 improves the outcome from c_4 winning to c_2, whereas adding arbitrary many copies of voter 1 leaves c_4 as the winner — as required. □