Strategic Merging of Project Proposals in Participatory Budgeting

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ABSTRACT

Background: In approval-based participatory budgeting, project proposers may strategically merge proposals into bundles, affecting both which projects are selected and how outcomes score on welfare and fairness.

Objectives and Research Questions: We study PB-induced merging games and ask how the choice of a voting rule shapes incentives to merge: when are singleton coalitions stable, when are nontrivial merges required for stability, and how hard is it to find a stable coalitional structure?

Methods: We formalize merging as a coalitional game over project proposers and analyze stability under standard PB rules, proving rule-dependent structure theorems and existence results. For rules where stability may require merging, we complement the theory with complexity results and experiments measuring prevalence and impact of nontrivial merges.

Results: Stable coalition structures always exist. Under AV/cost, Phragmén, and MES-Appr, singleton coalitions are stable, while under BasicAV and MES-Cost stability can require nontrivial merges. For the latter two rules, we establish hardness results for computing stable structures and empirically quantify how often such merges occur and how they affect welfare and fairness.

Conclusions: Our results identify a sharp qualitative split among PB rules: some guarantee stable independence, whereas others may incentivize merging and pose computational challenges for stability verification, with measurable welfare and fairness consequences.

KEYWORDS

Participatory Budgeting, Stability, Fairness

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1 INTRODUCTION

Participatory budgeting is a process where members of a given society—typically people living in some city or under another local jurisdiction—vote on which projects to implement, among those that they submitted for consideration. Each project has some cost and the local government provides some budget to spend. The goal of participatory budgeting is to let people decide what is truly important for them, but also to educate them about the intricacies of preparing public projects and analyzing their costs. Yet, as in many cases where money is involved, such beautiful ideas sometimes get corrupted. For example, consider the Polish municipality of Wieliczka. The results of their two PB elections (for municipality-wide projects), for the years 2022 and 2023, include the following two projects (the same in both years, except for minor changes in costs and that the first one also includes Janowice village in 2023):

(1) Safety upgrade in the villages of Mietniów, Pawlikowice, Chorągwica, Grajów, Dobranowice, Jankówka, Raciborsko, Lednica Górna, Podstolice, Gorzków, Janowice.
(2) Improvements in the villages of Brzegi, Byszyce, Czarnochowice, Grabie, Kokotów, Mała Wieś, Strumiany, Sułków, Śledziejowice, Węgrzce Wielkie, Zabawa.

These projects do not appear to be submitted by members of the general public, but rather look like the effect of coordination and merging of ideas provided by the heads of the respective villages. Although this may help these villages supplement their local budgets, it also blurs the usual PB ideal of many specific, independently proposed projects being evaluated directly by the public.

Our main goal is to analyze the possibility and consequences of project merging in PB elections. To this end, we define coalitional games where projects are the players and coalitions represent merged proposals. A coalition is successful if the joint project that it represents gets funded; we focus on deviations by proposers of projects that are not funded under the current structure (so a deviation is a way to turn previously unfunded projects into funded ones). In our analysis, we focus on the classic greedy approval rule (which we call BasicAV) and on the Method of Equal Shares (MES), but occasionally we also consider a few other rules.

We ask the following three questions: (1) Under which rules is the existence of stable coalitions guaranteed? (2) When is the coalition of singletons stable and how computationally challenging is recognizing such situations? (3) How does forming stable coalitions affect voter-oriented outcome measures (including cost-based welfare and standard fairness-oriented measures such as

Chamberlin–Courant coverage and Nash welfare)? Our analysis is both theoretical and experimental.

Intuitively, the voting rule used in a PB election should have strong impact on proposers' incentives to merge projects. Indeed, one reason why merging is so ubiquitous in Wieliczka's PB is that—like many other municipalities—it uses the classic greedy rule: This rule considers projects from the most to the least supported one, funding each that fits into the remaining budget at the time of its consideration. Under this rule, merging can be beneficial because increased support may outweigh the risk of not fitting into the budget (especially as this risk can often be predicted from past experience).

So would Wieliczka get different results if it used a different rule? We cannot say for sure, but when in 2023 it applied MES in its experimental ecology-oriented "Green Million" PB, the results were very different: 64 projects were submitted, of which 30 were funded, none of which appeared to be a result of merging.[1] For example, the following three projects were among the funded ones:

(1) Purchase of ten phototraps against littering.
(2) Breeding boxes for the common swifts.
(3) Arranging the slope by the Gorzków Primary School.

They are much more specific than the ones submitted in their usual PB elections. One might think that this is simply because MES is a proportional rule, and so merging is not beneficial. Our results show that the situation is more subtle:

> We find that both under BasicAV and under MES, merging can be beneficial and the coalition of singletons need not be stable. Yet, experiments show that singleton coalitions are far more often stable under MES than under the greedy rule (specifically, for the MES variant with cost utilities).

Interestingly, there are also proportional rules, such as Phragmén, or MES with approval utilities, as well as nonproportional ones, such as AV/Cost, under which merging is never beneficial. In particular, this pinpoints a strong difference between the two variants of the Method of Equal Shares. This is in line with the results of Faliszewski et al. [10], who found that these two variants incentivize proposers to price their projects at different levels.

Additionally, our experiments provide insights into how merging affects welfare- and fairness-oriented measures under MES with cost utilities and under BasicAV. In particular, MES appears substantially less sensitive to merging than BasicAV, both in terms of changes in cost-based welfare and in terms of equality among voters.

Related Work. Our work is closely related to several lines of research. First, *coalition formation* has been of crucial importance [20]. Within social choice theory, one area concerned with forming coalitions focuses on *hedonic games*, introduced by Dreze and Greenberg [6]. There, the task of the designer is to partition the agents into coalitions in such a way as to ensure the stability of the proposed partition (assuming that the agents have preferences over what groups they belong to). This problem has been extensively studied from the perspective of computational complexity, including problems such as checking the existence of core-stable paritions [3] or Nash-stable ones [12]. See the work of Aziz and Savani [1] for an overview of research on hedonic games.

Faliszewski et al. [10] recently studied strategic behavior of project proposers in participatory budgeting from the perspective of strategic cost selection. Moreover, Faliszewski et al. [11] investigated how project proposers who have a number of potential proposals can strategically choose which of them they should put forward. In both of those contributions, contrary to the scenario we consider here, project proposers act non-cooperatively.

Another line of research that is closely related to the game-theoretic approach to participatory budgeting is *strategic candidacy*, initiated by Dutta et al. [7]; see also, e.g., [4, 8, 13, 17–19]. There, candidates taking part in an election can choose to abstain in order to improve the election outcome for themselves. This is similar in spirit to coalition formation among candidates, in that candidates strategically take actions to influence the outcome in their favor.

Finally, questions of stability under strategic grouping have also been studied in related allocation settings; for example, Young and Balinski [2] study stability and coalition-like phenomena in apportionment systems.

2 PRELIMINARIES

A *participatory budgeting election* (an election, for short) is a tuple $E = (P, V, B)$, where $P = \{p_1, \dots, p_m\}$ is a set of *projects*, $V = \{v_1, \dots v_n\}$ is a set of *voters*, and $B \in \mathbb{R}_+$ is a *budget*. Furthermore, each project $p \in P$ has its associated positive *cost*, i.e., $\mathrm{cost}(p) > 0$. For a set of projects $W \subseteq P$, we say that W is *feasible* if the sum of their costs does not exceed the budget, i.e., $\sum_{p \in W} \mathrm{cost}(p) \le B$. We also say that a feasible set W is *exhaustive* if no proper superset of W is feasible. Each voter $v \in V$ has a set $A(v) \subseteq P$ of projects that v *approves*. For a project $p \in P$, we write $S(p) = \{v \in V \mid p \in A(v)\}$ for the set of its supporters.

PB Voting Rules. A *voting rule* (a *rule*, for short) is a function F that, given an election E, selects a feasible set of projects (an *outcome*) for this election. Below we define the rules that we focus on. All of them start with an empty outcome W and proceed sequentially, at each step adding a new project to W. Some of the rules require tie-breaking to make them resolute; in such cases we assume a fixed lexicographic tie-breaking order $\succ$.

BasicAV. At each step, a project p with $\mathrm{cost}(p) \le B - \mathrm{cost}(W)$ maximizing $|S(p)|$ is chosen, until W is exhaustive (or neither of the remaining projects fits in the budget). This rule is also known as *greedy approval voting*.

AV/Cost. At each step, a project p with $\mathrm{cost}(p) \le B - \mathrm{cost}(W)$ maximizing the score of $|S(p)|/\mathrm{cost}(p)$ is chosen, until W is exhaustive (or neither of the remaining projects fits in the budget). This rule is also known as *greedy approval-per-cost voting*.

Phragmén [5, 14]. Each voter v maintains personal funds b_v, initially equal to 0. Voters receive virtual money continuously, at a constant rate. As soon as for some project p it holds that $\sum_{v \in S(p)} b_v = \mathrm{cost}(p)$ and $\mathrm{cost}(p) \le B - \mathrm{cost}(W)$, project p is chosen and voters' personal funds are updated (b_v is set to 0 for each $v \in S(p)$). This process continues

[1] This comparison to their usual PB elections is not fully adequate, as in the "Green Million" one could not submit a project with a cost higher than 10% of the total budget, whereas in regular PB elections in Wieliczka the limit is 50%. Still, if proposers acted as usual, we should expect to see only 10 projects funded, not 30.

until W is exhaustive (or neither of the remaining projects fits in the budget).

MES-Appr [15, 16]. Each voter v is given an endowment $b_v \in \mathbb{R}_+$ that initially equals B/n. A project p is called ρ-*affordable* if its cost can be covered by its supporters so that none of them pays more than ρ units, i.e.,

$$\sum_{v \in S(p)} \min(b_v, \rho) = \text{cost}(p).$$

At each step, among all affordable projects the rule selects one with minimum ρ (ties are broken by $\succ$), and updates endowments with b_v set to $\max(0, b_v - \rho)$ for each $v \in S(p)$. This process continues until there are no affordable projects.

MES-Cost. This rule is identical to MES-Appr except that project p is ρ-affordable if

$$\sum_{v \in S(p)} \min(b_v, \rho \cdot \text{cost}(p)) = \text{cost}(p).$$

The rule selects a project with minimum ρ and sets b_v to $\max(0, b_v - \rho \cdot \text{cost}(p))$ for each $v \in S(p)$.

3 THE MODEL OF COALITIONAL ELECTIONS

Given an election $E = (P, V, B)$, we assume that each proposer has exactly one project to submit. Then, we consider the possibility that proposers merge their projects into *coalitions* (bundles), which then compete as single proposals. Formally, we consider a partition $\mathcal{P} = \{\mathcal{P}_1, \dots, \mathcal{P}_\ell\}$ of P, where each $\mathcal{P}_i$ in $\mathcal{P}$ is a coalition.

Compatibility Graph. We assume that not all sets of projects can be merged into a coherent proposal. To model feasibility constraints, we introduce a *compatibility graph* $G = (P, \mathcal{R})$ whose vertices are projects and edges represent pairwise compatibility. In our baseline model, we require that each coalition forms a connected component within G. This assumption is deliberately simple and ensures that feasibility constraints can be checked efficiently; later, we discuss that stricter notions (e.g., requiring coalitions to form cliques or to have bounded diameter) may better capture tight compatibility in practice.

Coalitional elections/structures. Given a partition $\mathcal{P}$, a *coalitional election* (or, abusing the notation, but following a more game-theoretic language, a *coalitional structure*) is a PB election, where each of the projects is a coalition. We assume that within each coalition both the set of supporters and the costs are combined. So, given a coalitional election $E_{\mathcal{P}} = (\mathcal{P}, V, B)$, for every $\mathcal{P}_i \in \mathcal{P}$ we have:

(1) $\text{cost}(\mathcal{P}_i) = \sum_{p \in \mathcal{P}_i} \text{cost}(p)$,
(2) $S(\mathcal{P}_i) = \bigcup_{p \in \mathcal{P}_i} S(p)$,
(3) projects in $\mathcal{P}_i$ form a connected component of G.

Item (2) is a baseline *approval-transfer* assumption: a voter supports a merged project if they support at least one of the projects that it includes. This captures a simple "bundling" effect (union of supporters) but may overestimate support when voters are cost-sensitive.

By $\mathcal{P}(p)$ we denote the coalition in $\mathcal{P}$ to which project p belongs. Given a voting rule F, we say that a project $p \in P$ is *satisfied* in $E_{\mathcal{P}}$ if $\mathcal{P}(p)$ is selected by F in $E_{\mathcal{P}}$. We assume that the tie-breaking order in $E_{\mathcal{P}}$ follows the ordering of the best project in each coalition with respect to $\succ$. We also assume that in a (coalitional) PB election no project costs more than B.

Independent coalitional structure. We will often consider the special case where each project competes on its own, i.e., where each project $p \in P$ forms the singleton coalition $\{p\}$. We call this the *independent coalitional structure* (ICS).

Definition 3.1 (Independent Coalitional Structure, ICS). We say that a coalitional election $E_{\mathcal{P}}$ is independent if, for every coalition $\mathcal{P}_i \in \mathcal{P}$, $|\mathcal{P}_i| = 1$.

Stable coalitional structures. Take a coalitional structure $E_{\mathcal{P}} = (\mathcal{P}, V, B)$ and a set of *deviating* projects $D \subseteq P$. A coalitional structure $E_{\mathcal{P}'} = (\mathcal{P}', V, B)$ is a deviation by D if the projects in D form one coalition in $\mathcal{P}'$ and, for every pair of projects $p, p' \notin D$, the two projects belong to the same coalition in $\mathcal{P}'$ if and only if they belonged to the same coalition in $\mathcal{P}$ (i.e., the partition remains unchanged apart from merging D). Given a voting rule F, we say that such a deviation is *valid* if every project $p \in D$ is satisfied in $E_{\mathcal{P}'}$ but not in $E_{\mathcal{P}}$. Finally, we say that $\mathcal{P}$ is *stable* if there are no valid deviations by any set D.

In this formulation, a deviation is a coordinated action by proposers of projects that are not funded under the current structure. This captures a common strategic incentive: unsuccessful proposers may bundle to increase the chance that their (merged) proposal is selected. We view this as a conservative baseline; extensions that allow funded projects to merge with unfunded ones (e.g., with side payments) are a natural direction for future work.

We illustrate the setting with the following example.

Example 3.1. *Take an election with the set of projects* $\{p_1, p_2, p_3, p_4\}$ *and the set of voters* $\{v_1, v_2, v_3\}$. *Let* $S(p_1) = \{v_1, v_3\}$, $S(p_2) = \{v_1\}$, $S(p_3) = \{v_1, v_2\}$, *and* $S(p_4) = \{v_1, v_2, v_3\}$. *Further, set the project costs so that* $\text{cost}(p_1) = \text{cost}(p_2) = \text{cost}(p_3) = 4$ *while* $\text{cost}(p_4) = 12$. *Moreover, let* $B = 12$ *and let* p_4 *be last in the tie-breaking order. Let the compatibility graph have two connected components: a path* $p_1 - p_2 - p_3$ *and an isolated vertex* p_4. *Let the voting rule be BasicAV.*

In this election the only funded project is p_4, *as it has the largest set of supporters and its cost amounts to the entire budget. However, in the coalitional structure* $\{\{p_1, p_2, p_3\}, \{p_4\}\}$, *the first coalition is funded (it has the same support as* p_4 *but is preferred by tie-breaking), and so the ICS is not stable.*

4 EXISTENCE OF STABLE COALITIONAL STRUCTURES

We first ask whether it is ever beneficial for groups of projects to merge, i.e., to deviate from the independent coalitional structure. For some rules, such as AV/Cost, Phragmén, and MES-Appr, ICS is always stable and the proposers have no incentive to merge their projects. For others, such as BasicAV and MES-Cost, ICS may fail to be stable. In the former case, the intuition is that the rules treat support relative to cost in a way that makes bundling unattractive: merging increases cost in the same "currency" in which the rule evaluates projects.

Towards this result, we use the following technical lemma, which allows us to reason about Phragmén analogously to AV/Cost.

Lemma 4.1. *If project* p *is selected in round* i *of the Phragmén procedure, then it maximizes*

$$\frac{|S(p)|}{\text{cost}(p) - \sum_{v \in S(p)} b_v^i},$$

where b_v^i is the endowment of voter v at the beginning of round i.

Proof. This follows directly from the continuous-time definition: The first project to reach its cost threshold is the one with the smallest additional time required, which corresponds to maximizing the displayed expression. □

Theorem 4.1. *For AV/Cost, Phragmén, and MES-Appr, ICS is always stable.*

Proof. We show that no coalition consisting only of projects that are unfunded under ICS can form a valid deviation.

AV/Cost. Let C be any coalition of projects that are not funded under ICS. Let $p^\star \in C$ maximize $\frac{|S(p)|}{\text{cost}(p)}$ (breaking ties by $\succ$). Since $S(C) = \bigcup_{p \in C} S(p)$ and $\text{cost}(C) = \sum_{p \in C} \text{cost}(p)$, we have $\frac{|S(C)|}{\text{cost}(C)} \leq \frac{|S(p^\star)|}{\text{cost}(p^\star)}$..
Thus C cannot outrank $p^\star$ by the AV/Cost score, and so C cannot be selected while all of its members are not.

Phragmén. Fix a round i with current endowments $(b_v^i)_{v \in V}$. By Lemma 4.1, Phragmén selects a project maximizing

$$\frac{|S(p)|}{\text{cost}(p) - \sum_{v \in S(p)} b_v^i}$$

among those that still fit. Consider any coalition C consisting only of projects not funded under ICS. In that round, the coalition would have the "score" of

$$\frac{|S(C)|}{\text{cost}(C) - \sum_{v \in S(C)} b_v^i}.$$

There exists a project $p^\star \in C$ whose score is at least that of the coalition (by the same averaging argument as for AV/-Cost, applied to the denominator with endowments). Hence C cannot become newly selected ahead of all its members, and so it cannot form a valid deviation.

MES-Appr. Fix a round with current endowments $(b_v)_{v \in V}$. If a coalition C is ρ-affordable, i.e., $\sum_{v \in S(C)} \min(b_v, \rho) = \text{cost}(C)$, then for every $p \in C$ we also have $\sum_{v \in S(p)} \min(b_v, \rho) \geq \text{cost}(p)$, because $S(p) \subseteq S(C)$ and $\text{cost}(p) \leq \text{cost}(C)$. Thus each constituent project is ρ-affordable as well. Since MES-Appr selects an affordable project with minimum ρ, a coalition of initially unfunded projects cannot all become funded through merging.

□

BasicAV and MES-Cost fail to guarantee that ICS is stable.

Theorem 4.2. *For BasicAV and MES-Cost, ICS is not guaranteed to be stable.*

Proof. We consider our rules one by one.

BasicAV. Consider projects $P = \{p_1, p_2, p_3, p_4\}$ with disjoint supporter sets, such that $|S(p_1)| = 2$ and $|S(p_2)| = |S(p_3)| = |S(p_4)| = 1$. Let $\text{cost}(p_1) = 3$, $\text{cost}(p_2) = \text{cost}(p_3) = \text{cost}(p_4) = 1$, and $B = 3$. Under ICS, only $\{p_1\}$ is funded. However, the coalition $\{p_2, p_3, p_4\}$ has support 3 and cost 3, so it is funded, yielding a valid deviation.

MES-Cost. Consider $P = \{p_1, p_2, p_3\}$, $V = \{v_1, \dots, v_4\}$, $B = 4$, $\text{cost}(p_1) = \text{cost}(p_2) = 2$, and $\text{cost}(p_3) = 3$. Let

$$S(p_1) = \{v_1, v_2\}, \qquad S(p_2) = \{v_3, v_4\}, \qquad S(p_3) = \{v_1, v_2, v_3\}.$$

Thus, p_1 and p_2 have two supporters each, whereas p_3 has three. Under the ICS, at the first step of MES-Cost we have that p_1 and p_2 are 1/2-affordable, while p_3 is 1/3-affordable; hence p_3 (with smaller affordability parameter ρ) is selected first, and no further project is affordable afterwards.
Now consider the coalitional structure with $C_1 = \{p_1, p_2\}$ and $C_2 = \{p_3\}$. Then C_1 is 1/4-affordable, while C_2 remains 1/3-affordable, so C_1 is funded instead, yielding a profitable deviation. Therefore, ICS is not stable in this instance.

□

Nonetheless, for both BasicAV and MES-Cost there always is a stable coalitional structure.

Theorem 4.3. *BasicAV and MES-Cost always admit a stable coalitional structure.*

Proof. We argue separately for the two rules.

BasicAV. Initialize $R := P$ and $B := B$, and build a partition $\mathcal{P}$ greedily. While there exists a coalition $C \subseteq R$ contained in a connected component of $G[R]$ with $\text{cost}(C) \leq B$, choose

$$C^\star \in \arg\max\{|S(C)| : C \subseteq R,\ C \text{ connected in } G[R],\ \text{cost}(C) \leq B\}$$

breaking ties by $\succ$, add $C^\star$ to $\mathcal{P}$, and set $R := R \setminus C^\star$, $B := B - \text{cost}(C^\star)$. When no such coalition exists, put each project in R in a singleton coalition.
By construction, BasicAV on $E_{\mathcal{P}}$ selects the chosen coalitions in this order. When the process stops, no coalition of the remaining (unfunded) projects fits the residual budget B, so no unfunded set can merge into a coalition that would be selected. Hence $\mathcal{P}$ is stable.

MES-Cost. Iteratively fix, among the remaining projects, a feasible coalition that minimizes the affordability parameter ρ (breaking ties by $\succ$), until none exists; keep the rest as singletons. Since each step selects the most affordable available coalition, no set of initially unfunded projects can deviate to become selected. Thus the resulting structure is stable.

□

The results of this section call for two comments. First, our PB voting rules form two groups: The first one consists of BasicAV and MES-Cost, for which ICS is not guaranteed to be stable (and we only have an exponential-time construction of a stable structure). The second one consists of AV/Cost, Phragmén, and MES-Appr, for which ICS *is* always stable. In particular, this differentiates the two variants of the Method of Equal Shares, reinforcing the results of Faliszewski et al. [10], who found a similar separation in the context of choosing project costs.

Second, while Theorem 4.3 guarantees the existence of a stable coalitional structure, it does not imply that one can be found efficiently, and the stable structure may differ substantially from the original election. Hence, we next study the computational complexity of testing whether ICS is stable.

5 COMPUTATIONAL ASPECTS OF STABLE COALITIONAL STRUCTURES

For BasicAV and MES-Cost, the independent coalitional structure is not guaranteed to be stable. We therefore study the computational complexity of checking stability-related questions. Intuitively, these questions capture whether profitable merging opportunities can be identified efficiently, which is relevant both for predicting strategic behavior and for auditing a rule's vulnerability.

First, we consider checking whether the independent coalitional structure is stable.

Stability of Independent Elections	
Input:	Election $E = (P, V, B)$ and voting rule f.
Question:	Is the independent coalitional structure for E stable with respect to f?

Second, we analyze whether a given coalition of projects can be funded in *some* coalitional structure.

Possible Funding	
Input:	Election $E = (P, V, B)$, voting rule f, and coalition $\mathcal{P}^* \subseteq P$.
Question:	Is there a coalitional structure $E_{\mathcal{P}}$ in which the coalition $\mathcal{P}^*$ is funded?

Finally, we consider the question whether a coalition is funded in *every* coalitional structure.

Necessary Funding	
Input:	Election $E = (P, V, B)$, voting rule f, and coalition $\mathcal{P}^* \subseteq P$.
Question:	Is $\mathcal{P}^*$ funded in every coalitional structure $E_{\mathcal{P}}$ that includes $\mathcal{P}^*$?

Even though each of our voting rules guarantees the existence of a stable coalitional structure, deciding whether a particular coalition can be funded (or must be funded) can still be computationally hard.

Proposition 5.1. *Possible Funding is NP-complete for BasicAV and MES-Cost.*

Proof. Membership in NP is immediate: a certificate is a coalitional structure, and winners under BasicAV and MES-Cost can be computed in polynomial time.

For NP-hardness, we reduce from Exact Cover by 3-Sets (X3C): given a universe X with $|X| = 3t$ and a family S of 3-element subsets of X, decide whether there exists $S^* \subseteq S$ that covers each element of X exactly once.

BasicAV. Given an X3C instance (X, S), we construct an election as follows. For each $x \in X$ there is a voter v_x, and there is one additional voter v_e. For each set $s \in S$ we create a project p_s (a *set project*) with cost $c(p_s) = 3$, approved by exactly the voters $\{v_x : x \in s\}$. We also create two further projects: a project p^* with cost $c(p^*) = B$ approved by all voters $\{v_x : x \in X\} \cup \{v_e\}$, and a *verifier* project p^+ with cost $c(p^+) = 1$ approved only by v_e. Finally, we set $B = 3t + 1$ and assume that p^* is ranked first in the tie-breaking order.

We claim that $\{p^+\}$ is funded in some coalitional structure if and only if the X3C instance is positive. If there exists an exact cover S^* of size t, consider the coalition $\mathcal{P}^* = \{p^+\} \cup \{p_s : s \in S^*\}$. This coalition costs $3t + 1 = B$ and is approved by all voters (every v_x approves exactly one set project, and v_e approves p^+). Thus, under BasicAV, $\mathcal{P}^*$ ties p^* in approval score and (by tie-breaking) can be made the funded coalition in an appropriate coalitional structure, which in turn funds p^+. Conversely, if there is no exact cover, then any coalition containing p^+ and costing at most B can include at most t set projects, so it leaves at least one element voter v_x unrepresented and hence has strictly fewer approvals than p^*; therefore p^+ cannot be funded in any coalitional structure.

MES-Cost. The NP-hardness for MES-Cost follows by an analogous encoding of X3C. (We use the same high-level idea: a "grand" project p^* that is selected in the independent election, and a verifier project p^+ that can be funded only if a coalition corresponding to an exact cover defeats p^* under MES-Cost. The construction ensures that this can happen if and only if an exact cover exists.) ◻

Proposition 5.2. *Necessary Funding is coNP-complete for BasicAV and MES-Cost.*

Proof. The problem is in coNP: if a coalition $\mathcal{P}^*$ is *not* funded in every coalitional structure, then a certificate is a coalitional structure witnessing this, which can be verified in polynomial time.

For coNP-hardness, we use the same constructions as in the proof of Proposition 5.1, but now ask whether $\{p^*\}$ is funded in every coalitional structure. ◻

We next turn to the problem of deciding if the independent coalitional structure is stable (note that hardness results for this problem also imply hardness of a more general problem, where we ask if a given coalitional structure is stable). A straightforward approach is to test all feasible deviations and check whether any deviation is profitable. This yields tractability whenever the number of feasible deviations is polynomially bounded.

Proposition 5.3. *For BasicAV and MES-Cost, Stability of Independent Elections is polynomial-time solvable if the compatibility graph has a polynomially bounded number of connected components.*

Proof. Given the compatibility graph $G = (P, E)$, we can compute its connected components in polynomial time. Under our model, any deviating coalition must be contained in a single connected component, and if the number of components is polynomially bounded, then we can iterate over all of them. For each component C, we can check in polynomial time (by a single run of BasicAV or MES-Cost) whether merging the projects in C is a profitable deviation from the independent coalitional structure. Thus, we can decide stability in polynomial time. ◻

As a corollary, checking ICS stability is tractable, e.g., for compatibility graphs that are collections of paths and cycles. However, the problem becomes significantly more challenging even for sligthly more involved graphs. Indeed, we show that Stability of Independent Elections is already hard even on caterpillar graphs.

Definition 5.1. A graph $G = (V, E)$ is a *caterpillar* if G is a tree and contains a path P such that every vertex in V has distance at most 1 from P.

Theorem 5.4. *For BasicAV and MES-Cost, Stability of Independent Elections is coNP-complete even if the compatibility graph is a caterpillar.*

Proof. Membership in coNP follows because a "no" instance (instability) has a polynomially verifiable certificate: a connected set of projects that forms a profitable deviation from the independent coalitional structure.

For coNP-hardness, we reduce from the complement of Restricted Exact Cover by 3-Sets (RX3C), the restriction of X3C where $|S| = 3t$. Given an RX3C instance (X, S) with $|X| = 3t$, we construct an election (the *encoding* of the instance) as follows. For each $x \in X$ there is an *element voter* v_x, and there is an additional voter v_e. For each set $s \in S$ we create a *subset project* p_s with cost $c(p_s) = 3$, approved by $\{v_x : x \in s\} \cup \{v_e\}$. We also create a project p^* with cost $c(p^*) = 3t + 1$ approved by all voters, and $|S| + 1$ *caterpillar projects* $q_1, \dots, q_{|S|+1}$, each approved only by v_e and each with cost $c(q_i) = \frac{1}{|S|+1}$. We set $B = 3t + 1$ and place p^* last in the tie-breaking order.

The compatibility graph is a caterpillar: the caterpillar projects form the spine (a path), and every other project (p^* and each p_s) is attached as a leaf to a distinct spine vertex.

In the independent coalitional structure, only p^* is funded: under BasicAV, it has strictly more approvals than any other singleton project and it exhausts the budget; under MES-Cost, it is considered first (by its score) and exhausts the budget as well.

If the RX3C instance is positive, let $S^* \subseteq S$ be an exact cover of size t. Consider the coalition consisting of the t subset projects corresponding to S^* together with all caterpillar projects. This coalition costs $3t + 1 = B$ and is approved by all voters, and it is considered before p^* by tie-breaking; hence it forms a profitable deviation, so the independent coalitional structure is not stable.

Conversely, if the RX3C instance is negative, then no collection of subset projects of total cost at most $3t$ can cover all element voters, so any feasible coalition different from $\{p^*\}$ has strictly fewer approvals than p^* (under BasicAV) and cannot beat p^* under MES-Cost by construction. Thus, there is no profitable deviation, and the independent coalitional structure is stable. □

6 PRICE OF MERGING

Given that project proposers may have incentives to merge their submissions, it is important to understand how such behavior affects the voters' welfare. To quantify this effect, we follow the common approach of *cost utilities* in participatory budgeting: a voter's utility from an outcome is the total cost of funded projects that they approve.

Definition 6.1 (Utility and Social Welfare). Let $E_{\mathcal{P}} = (\mathcal{P}, V, B)$ be a coalitional structure, and let $W \subseteq \mathcal{P}$ be a feasible set of funded coalitions. For a voter $v \in V$, we define

$$u_v(E_{\mathcal{P}}, W) \;=\; \textstyle\sum_{C \in W} \sum_{p \in C \cap A(v)} \mathrm{cost}(p).$$

The *social welfare* of W in $E_{\mathcal{P}}$ is

$$\mathbf{SW}_{E_{\mathcal{P}}}(W) \;=\; \textstyle\sum_{v \in V} u_v(E_{\mathcal{P}}, W).$$

We next define a measure of how coalition formation changes welfare, by comparing the welfare achieved under a given coalitional structure to that achieved under the independent coalitional structure.

Definition 6.2 (Price of Merging). Let E be a PB election, let f be a voting rule, and let $E_{\mathcal{P}}$ be a coalitional structure of E. Let $E_{\mathcal{P}^*}$ denote the independent coalitional structure. We denote

$$x = \mathbf{SW}_{E_{\mathcal{P}^*}}\big(f(E_{\mathcal{P}^*})\big) \qquad \text{and} \qquad y = \mathbf{SW}_{E_{\mathcal{P}}}\big(f(E_{\mathcal{P}})\big).$$

Then, the *price of merging* is

$$\mathrm{PoM}(E, f, E_{\mathcal{P}}) \;=\; \tfrac{x-y}{\min(x,y)}.$$

With this definition, positive values indicate welfare loss due to merging (i.e., $y < x$), while negative values indicate welfare gains.

Proposition 6.1. *For BasicAV and MES-Cost, the price of merging can be arbitrarily large (positive) or arbitrarily small (negative).*

Proof. We first show that PoM can be arbitrarily large for both rules. Consider an election with projects $P = \{p_1, p_2, p_3\}$, budget $B = 1$, and two voters $V = \{v_1, v_2\}$ with $A(v_1) = \{p_1, p_2\}$ and $A(v_2) = \{p_1, p_3\}$. Let $\mathrm{cost}(p_1) = 1$ and $\mathrm{cost}(p_2) = \mathrm{cost}(p_3) = \epsilon$, and fix a tie-breaking order $p_2 \succ p_3 \succ p_1$.

In the independent coalitional structure, both BasicAV and MES-Cost select $\{p_1\}$: it has the largest support and exhausts the budget. Now consider the coalitional structure $\mathcal{P} = \big\{\{p_1\}, \{p_2, p_3\}\big\}$. Both rules select the coalition $\{p_2, p_3\}$, since it has the same approval support as $\{p_1\}$ but is preferred by tie-breaking, and it is feasible (cost $2\epsilon \le 1$).

Under $E_{\mathcal{P}^*}$, the welfare is $x = \mathbf{SW}_{E_{\mathcal{P}^*}}(\{p_1\}) = 2$, whereas under $E_{\mathcal{P}}$, the welfare is

$$y = \mathbf{SW}_{E_{\mathcal{P}}}(\{\{p_2, p_3\}\}) = 4\epsilon.$$

Hence,

$$\mathrm{PoM}(E, f, E_{\mathcal{P}}) = \tfrac{2-4\epsilon}{4\epsilon} = \tfrac{1}{2\epsilon} - 1,$$

which tends to $+\infty$ as $\epsilon \longrightarrow 0$.

We now show that PoM can be arbitrarily small (i.e., arbitrarily negative).

BasicAV. Let $P = \{p_1, p_2, p_3, p_4\}$ and $B = 1$. Fix $\epsilon > 0$ and an integer $\gamma > 2$, and let $V = \{v_1, \dots, v_\gamma\}$. Assume the approval sets are such that

$$S(p_1) = \{v_1, \dots, v_{\gamma-1}\},\ S(p_2) = \{v_\gamma\},\ S(p_3) = V,\ S(p_4) = \{v_1, v_2\}.$$

Let costs be $\mathrm{cost}(p_1) = 1 - \epsilon$, $\mathrm{cost}(p_2) = \epsilon$, $\mathrm{cost}(p_3) = 3\epsilon$, and $\mathrm{cost}(p_4) = 1 - 3\epsilon$.

In the independent coalitional structure, BasicAV funds $\{p_3, p_4\}$ (feasible with total cost 1), yielding

$$x = \mathbf{SW}_{E_{\mathcal{P}^*}}(\{p_3, p_4\}) = \gamma \cdot 3\epsilon \;+\; 2 \cdot (1 - 3\epsilon) = 2 + (3\gamma - 6)\epsilon.$$

Now consider the coalitional structure $\mathcal{P} = \big\{\{p_1, p_2\}, \{p_3\}, \{p_4\}\big\}$. The coalition $\{p_1, p_2\}$ has γ approvals (all voters), which is strictly more than any other coalition in $\mathcal{P}$, and it is feasible with cost 1. Thus BasicAV funds $\{\{p_1, p_2\}\}$ and

$$y = \mathbf{SW}_{E_{\mathcal{P}}}(\{\{p_1, p_2\}\}) = (\gamma - 1) \cdot (1 - \epsilon) + 1 \cdot \epsilon = \gamma - 1 - (\gamma - 2)\epsilon.$$

As γ grows (and ϵ is small), we have $y \gg x$, so $\mathrm{PoM} = (x - y)/\min(x, y)$ becomes arbitrarily negative.

MES-Cost. Consider $P = \{p_1, p_2, p_3\}$, budget $B = 1$, and $n > 2$ voters $V = \{v_1, \ldots, v_n\}$. Let costs be $\text{cost}(p_1) = 1 - \epsilon$, $\text{cost}(p_2) = \epsilon$, and $\text{cost}(p_3) = \frac{2}{n}$. Assume

$$S(p_1) = \{v_1, \ldots, v_{n-1}\}, \quad S(p_2) = \{v_n\}, \quad S(p_3) = \{v_{n-1}, v_n\}.$$

In the independent coalitional structure, MES-Cost cannot fund p_1 as its supporters cannot cover its cost, so it selects p_3 (which has higher approval than p_2), and then stops because the relevant supporters run out of remaining budget. So, the funded set is $\{p_3\}$ and $x = \mathbf{SW}_{E_{\mathcal{P}^*}}(\{p_3\}) = 2 \cdot \frac{2}{n} = \frac{4}{n}$.

Now merge p_1 and p_2 into a single coalition of cost 1 supported by all voters. Then MES-Cost funds this merged project, yielding

$$y = \mathbf{SW}_{E_{\mathcal{P}}}(\{\{p_1, p_2\}\}) = (n-1)(1-\epsilon) + \epsilon.$$

Therefore,

$$\text{PoM}(E, f, E_{\mathcal{P}}) = \frac{\frac{4}{n} - \big((n-1)(1-\epsilon) + \epsilon\big)}{\min\!\Big(\frac{4}{n},\ (n-1)(1-\epsilon) + \epsilon\Big)},$$

which is arbitrarily negative as n increases (and ϵ is small). □

7 EXPERIMENTAL ANALYSIS

Finally, we examine the stability of BasicAV and MES-Cost on real-world PB data from PABULIB [9].

7.1 Pabulib Instances

We computed outcomes of BasicAV and MES-Cost across 434 approval-based instances in PABULIB with at most 20 projects (data collected in September 2025). We impose this restriction because checking stability is NP-hard in general and becomes quickly infeasible on larger instances.

7.2 β-Compatibility Graph

To instantiate the compatibility graph on real data, we generate edges based on the degree of *overlap in support* between projects. The intuition is that coalitions should represent cohesive proposals; we use overlap in supporters as a proxy for how naturally two projects fit together.

Formally, fix $\beta \in [0, 1]$ and a PB election $E = (P, V, B)$. For projects $p_i, p_j \in P$, we say that p_i and p_j are *β-compatible* if

$$\frac{|S(p_i) \cap S(p_j)|}{|S(p_i) \cup S(p_j)|} \ \geq\ \beta.$$

In the compatibility graph we consider, two projects are adjacent if and only if they are β-compatible.

Definition 7.1 (β-Compatibility Graph). For a PB election $E = (P, V, B)$, the *β-compatibility graph* is the graph $G = (P, \mathcal{R})$ where $\{p_i, p_j\} \in \mathcal{R}$ if and only if p_i and p_j are β-compatible.

7.3 ICS Stability

Our first experiment investigates how often is ICS unstable on real-world instances. Figure 1 reports the fraction of analyzed elections for which the ICS is stable, as a function of β. We observe that MES-Cost yields a substantially higher stability rate than BasicAV for small and moderate values of β. As expected, stability increases with β for both rules: Requiring larger overlap shrinks the set of feasible merges and thus reduces the scope for profitable deviations.

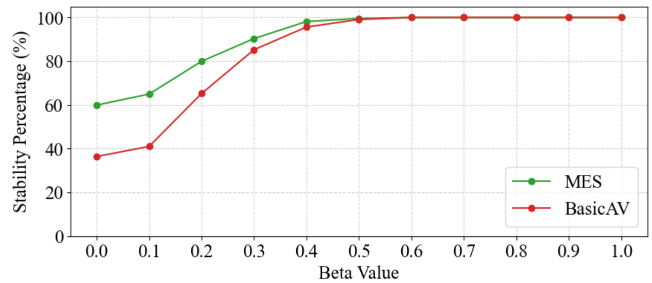


Figure 1: Stability of the independent coalitional structure (ICS) under MES-Cost and BasicAV. Each data point shows the percentage of analyzed instances for which the ICS is stable, assuming the β-compatibility graph for the given value of β.

The difference between MES-Cost and BasicAV also narrows as β grows, since the remaining potential merges are increasingly tight and therefore less likely to alter outcomes.

7.4 Welfare Properties

Our second experiment studies how deviations from ICS affect welfare and equality measures. Recall from Proposition 6.1 that for both BasicAV and MES-Cost the price of merging (PoM) with respect to social welfare can be arbitrarily large in either direction; here we assess how pronounced these effects are on real instances.

For each instance from Section 7.1, we enumerated all *successful* deviations consisting of a single merging coalition (i.e., one connected set of projects that merges, while all other projects remain singleton coalitions), and computed the resulting PoM values. This restriction ignores simultaneous merging by multiple coalitions. We adopt it because the space of full partitions into coalitions is prohibitively large; at the same time, it captures the impact of a single coordinated group of proposers.

Figure 2(a) shows, for each instance, the *maximum* PoM with respect to social welfare among all successful single-coalition deviations; instances are sorted by this value. We see that the maximal welfare loss tends to be larger under BasicAV than under MES-Cost.

The corresponding plots for the average and minimum PoM values exhibit similar patterns. In particular, fig. 3 shows a very for the average values of PoM for social welfare as in Figure 2(a). Again, we can observe that the differrences between BasicAV and MES-Cost stem from a very small number of instances. The remaining figures are deferred to the appendix.

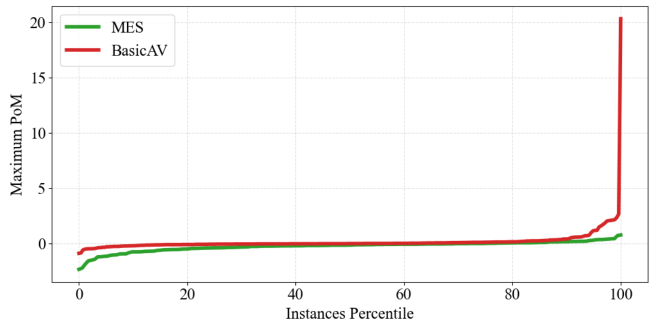
7.5 Equality Measures

Beyond total welfare, we examine how merging affects the *distribution* of benefits across voters. We use two standard proxies: the Chamberlin–Courant (CC) score (coverage) and Nash welfare.

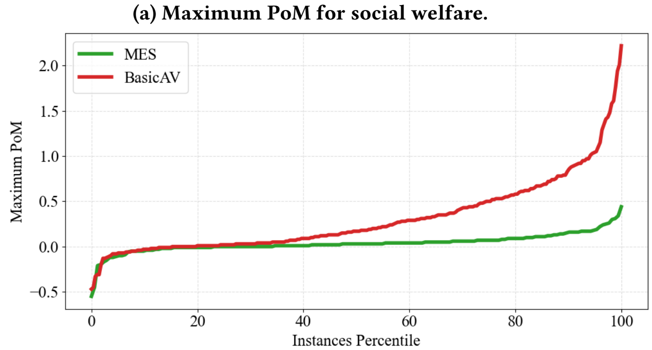
Definition 7.2 (Chamberlin–Courant Score)**.** Let $E_{\mathcal{P}} = (\mathcal{P}, V, B)$ be a coalitional structure and let $W \subseteq \mathcal{P}$ be a feasible set of funded coalitions. The *Chamberlin–Courant score* of W is

$$\mathbf{CC}_{E_{\mathcal{P}}}(W) \ = \ \Big|\{\, v \in V \mid A(v) \cap \textstyle\bigcup_{C \in W} C \neq \emptyset \,\}\Big|.$$

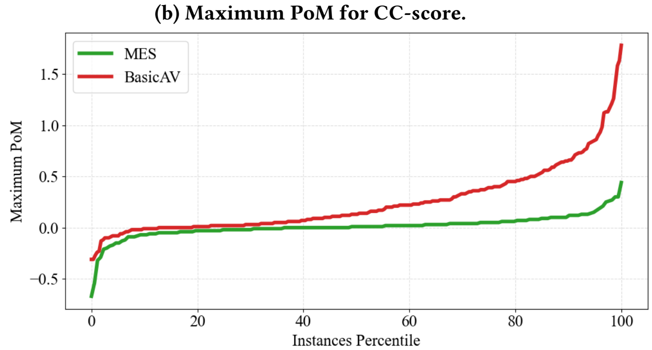
Definition 7.3 (Nash Welfare)**.** Let $E_{\mathcal{P}} = (\mathcal{P}, V, B)$ be a coalitional structure and let $W \subseteq \mathcal{P}$ be a feasible set of funded coalitions. The



(a) Maximum PoM for social welfare.



(b) Maximum PoM for CC-score.



(c) Maximum PoM for Nash welfare.

Figure 2: Maximum Price of Merging across successful single-coalition deviations (with $\beta = 0$), measured with respect to social welfare (top), Chamberlin–Courant score (middle), and Nash welfare (bottom). Instances are sorted increasingly by the plotted value. Red curves correspond to BasicAV and green curves to MES-Cost.

Nash welfare of W is

$$\mathbf{NW}_{E_{\mathcal{P}}}(W) = \sum_{v \in V : u_v(E_{\mathcal{P}}, W) > 0} \ln\big(u_v(E_{\mathcal{P}}, W)\big).$$

We define PoM for these measures analogously to Definition 6.2: for a measure $\Phi \in \{\mathbf{CC}, \mathbf{NW}\}$ we set

$$\mathrm{PoM}_{\Phi}(E, f, E_{\mathcal{P}}) = \frac{x - y}{\min(x, y)},$$

where $x = \Phi_{E_{\mathcal{P}^*}}(f(E_{\mathcal{P}^*}))$ and $y = \Phi_{E_{\mathcal{P}}}(f(E_{\mathcal{P}}))$, and $E_{\mathcal{P}^*}$ is the independent coalitional structure.

Figure 2(b) shows the maximum PoM values with respect to the CC-score, and Figure 2(c) shows the same for Nash welfare. In both cases, the upper tail is substantially larger for BasicAV than for

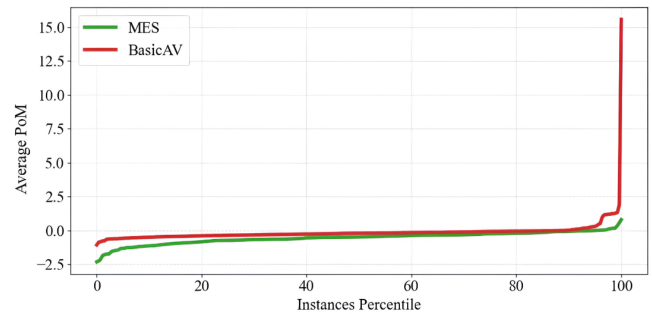


Figure 3: Average PoM across successful single-coalition deviations (with $\beta = 0$), measured with respect to social welfare. Instances are sorted increasingly by the plotted value. Red curves correspond to BasicAV and green curves to MES-Cost.

MES-Cost, suggesting that the strongest adverse effects of merging (in terms of coverage and Nash welfare) are more pronounced under BasicAV. The corresponding results for average and minimum PoM values are reported in the appendix.

8 CONCLUSION

In this work, we initiated the study of strategic merging by project proposers in participatory budgeting elections. For several PB voting rules, we show that the independent coalitional structure (ICS), in which all projects remain separate, is always stable. For the two rules for which ICS need not be stable, namely BasicAV and MES-Cost, we establish that even checking stability can be computationally intractable. At the same time, our experiments on real-world data indicate that MES-Cost yields stable ICS substantially more often than BasicAV, and that the adverse effects of merging—both in terms of social welfare and equality measures—are more pronounced under BasicAV.

Our results provide many opportunities for further research.

(1) In the notion of stability we propose we only take into account deviations by groups of proposers whose projects were not funded. It is plausible, however, that a successful proposer can choose to team up with an unsuccessful one, potentially asking for a partial transfer of utility.
(2) Our computational complexity results are mostly negative. It would be of high interest to mitigate this, for instance via parameterized algorithms. A natural parameter to consider is the number of projects (or coalitions) in the ICS.
(3) In our setting we assume proposers have full knowledge of the parameters of the game. It is plausible, however, that they might be uncertain, for example about voters' exact preferences or about whether voters would be willing to support a merged project. Taking this into account would be a compelling follow-up.
(4) In our empirical analysis we restrict merging when projects do not have enough supporters in common. More generally, one can expect that a voter might not be willing to support a merged project if only a small fraction of its cost comes from projects that the voter approves. Incorporating such cost-sensitivity would provide an additional, and arguably more realistic, angle on strategic behavior by proposers.

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