Costly Voting in the Hotelling-Downs Model

Guy Wolf
Technion
Haifa, Israel
guy.wolf@campus.technion.ac.il

Reshef Meir
Technion
Haifa, Israel
reshefm@technion.ac.il

ABSTRACT

Background: The Hotelling-Downs model of political competition for two candidates has a single equilibrium where candidates take the position of the median voter. This contrasts with candidate polarization we often see in practice.

Objectives and Research Questions: Can a simple variation of the model that allow voters to abstain, explain candidate polarization?

Methods: Theoretical analysis. We use simple simulations to show the magnitude of the effect under specific distributions.

Results: We characterize candidate equilibrium positions under a simple model costly voting, when either costs or voters' positions are uniform. Results show that as costs increase and participation decreases, candidates become more polarized.

Conclusions: In contrast to the standard Hotelling–Downs benchmark, even under the simplest voting-cost specifications equilibria need not coincide with the median voter. Candidate polarization is exacerbated both by polarization of the voting population (which is not surprising), and by voters' disengagement.

KEYWORDS

Social Choice, Hotelling game, Polarization, Abstention

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1 INTRODUCTION

A standard framework for modeling strategic positioning in two-candidate electoral competition is the *Hotelling–Downs model* [11, 19], in which two office-seeking candidates choose policy platforms to maximize vote share. Under full voter participation and single-peaked preferences, this setting yields convergence to the median voter, reflecting the logic of the *median voter theorem*, which implies that the median voter's ideal policy defeats any alternative in pairwise majority voting.

However, a growing body of empirical evidence suggests that real-world political actors frequently do not converge to the median position. In the United States, Ansolabehere et al. [3] and Lee et al. [22] document systematic departures from median convergence in U.S. House elections, while [23] finds similar patterns in the U.S. Senate. Moreover, Bafumi and Herron [5] shows that members of Congress tend to adopt positions that are more ideologically extreme than those of their constituencies. Comparable evidence has also been reported in other political systems: in European contexts, parties often appear more responsive to their own supporters than to the median voter [14], and related patterns have been documented in Australia as well [28]. Together, these findings indicate that the canonical convergence prediction may be empirically fragile.

One key difference between the benchmark Hotelling–Downs model and real-world elections is that the benchmark typically assumes universal turnout, whereas in practice, voters may abstain. Notably, voter participation has declined in many democracies over recent decades [17, 18], making it increasingly important to incorporate non-participation into models of electoral competition.

In this paper, we extend the Hotelling–Downs framework by introducing costly voting. Voters participate only when the relative utility gain from supporting their preferred candidate exceeds their individual cost of voting; otherwise, they abstain. This mechanism implies that turnout depends endogenously on the perceived policy difference between the candidates, and candidates must account not only for ideological proximity but also for the turnout incentives induced by their platform choices. Our model accommodates arbitrary distributions of voter ideal points and voting costs.

1.1 Related Work

Hotelling–Downs and variants. Due to the simplicity and interpretability of the Hotelling-Downs spatial model, a large literature has developed extensions and variations, often with the aim of getting more 'natural' equilibria where candidates diverge. Most extensions retain the assumption that voters prefer the closer candidate but vary the details of the model, e.g. to consider voters' uncertainty about candidates [20], candidates' uncertainty about voters [8, 32], relaxing equilibrium requirements [35], asymmetries between candidate favourability Aragones and Palfrey [4] and so on.

Abstention. The modeling of voter abstention has received considerable attention since the outset of formal political economy, beginning with the calculus of voting framework [31], as well as later "lazy bias" models [9, 13]. In either case, the fundamental assumption is that voters are more likely to participate *when they are more pivotal*, and the analysis focuses on the implications for equilibrium outcomes. This assumption is relaxed in [26], where voters consider various heuristics for their pivotality. Other papers, such as [7, 27], examine the effects of arbitrary or random participation on electoral outcomes under the Median rule.[1] However, none of the above models endogenize candidate responses to abstention; instead, candidate positions are taken as fixed.

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[1]In some simulations in [7] the authors sampled more voters closer to the edges of the interval (i.e. assuming 'extreme' voters are more active), but without any underlying utility model.

Some variations of the Hotelling–Downs model explicitly incorporate the possibility of voter abstention. In [16, 34], voters abstain whenever all candidates lie outside a fixed *attraction interval.* This structure allows abstention, but does not generate a gradual decline in participation as candidates become less attractive. Coughlin [8] considers probabilistic attraction models with a finite set of voters, in which each voter may abstain according to an independent probability function. It is worth noticing that existing abstention models only consider the distance to the nearest candidate, which may be reasonable in *economic competition* but not in *political competition.*[2]

Candidate polarization. Political scientists make a distinction between *mass polarization* (of the voting population) and *elite polarization* (sometimes associated with the candidates). One prominent debate is on which type polarization precedes, or even causes, the other [1, 15, 24].

Voter participation has been well documented to decline in many democracies over recent decades [17, 18]. At the same time, polarization has increased in several regions, including the United States [6] and Europe [30]. Several recent studies [2, 12] link higher political polarization to higher willingness to abstain, providing empirical evidence consistent with the idea that higher voting costs may contribute to increased polarization.

Prior theoretical work in computational social choice has modeled polarization in a variety of settings, such as committee selection [10], which sometime require complex definitions to measure. In the context of 2-candidate spatial competition, measuring polarization is trivial, and so we can isolate and test the effect of various components of the model, in particular voters' distribution and voting costs.

1.2 Our Contribution

We first analyze benchmark environments in which either voter ideal points or voting costs are uniformly distributed, deriving simple characterization for Nash equilibrium candidate positions. We then characterize more complex equilibrium conditions in the fully general case, allowing for arbitrary position and cost distributions.

In Section 4 we leverage the above characterizations via theoretical results and simulations, to support our main argument. I.e. that high costs (i.e. low turnout) drives candidate polarization.

2 MODEL

We build upon the original Hotelling-Downs model.

2.1 The Hotelling-Downs model

Consider a setting where a continuum of voters is uniformly distributed along the interval $[0, 1]$. There are two candidates, who play a one-shot game where in each stage they both choose a position in the voter interval. Each voter votes for the candidate closest to them, and each candidate attempts to maximize the amount of voters that vote for them. In case both voters choose the same position, each of them gets half of the votes. It is well known that the only equilibrium is when both candidates pick 0.5.

In fact, this is true in a more general model. Let V be the CDF of an arbitrary continuous distribution of voters on $[0, 1]$.

Claim 1 (See Downs [11]). *The only equilibria are where both candidates pick x_1, x_2 such that $V(x_1) = V(x_2) = \frac{1}{2}$.*

2.2 Partial participation

We consider a two-candidate Hotelling–Downs model on the policy interval $[0, 1]$. The key departure from the standard framework is the introduction of a cost-benefit analysis by voters.

In addition to their position, each voter draws an idiosyncratic voting cost $\kappa \in [0, 1]$. Voting costs are independently and identically distributed across voters at all locations according to a continuous distribution with cumulative distribution function K and density k.

Participation is determined by whether the voter's utility gain from voting, captured by the distance advantage of the preferred candidate, is larger than κ.

We emphasize that this utility gain is based on the simplified assumption that the voter treats herself as always pivotal (in contrast e.g. to the *Calculus of Voting* model). This assumption can be justified e.g. when the voter does not know the position of the other voters, or when voters are boundedly rational [26].

Candidates. Two candidates, indexed by $i \in \{1, 2\}$, simultaneously choose policy positions $c_i \in [0, 1]$ in a one-shot game. Candidates are fully informed about the distributions of voter ideal points and voting costs. For a voter located at x, the distance to candidate i is

$$d_i(x) \ = \ |x - c_i|.$$

Voter behavior. After observing (c_1, c_2), each voter decides whether to vote for one of the candidates or abstain. A voter votes for the candidate closest to her ideal point if and only if her gain (the difference in distances to the two candidates) is at least as large as her voting cost κ. Formally, a voter at location x votes for candidate 1 whenever

$$d_2(x) - d_1(x) \ \geq \ \kappa,$$

and votes for candidate 2 whenever

$$d_1(x) - d_2(x) \ \geq \ \kappa;$$

Otherwise, the voter abstains. The share of voters in x that vote for i is thus

$$p_i(x) = \begin{cases} K\big(d_{-i}(x) - d_i(x)\big), & \text{if } d_i(x) < d_{-i}(x), \\ 0, & \text{if } d_i(x) \geq d_{-i}(x). \end{cases}$$

Candidate Utilities. Candidates attempt to maximize their vote share out of all voters. The utility for each candidate is thus equal to the total amount of votes they get

$$u_i(c_1, c_2) \ = \ \int_0^1 p_i(x)\, v(x)\, dx.$$

We focus on the identification of pure-strategy Nash equilibria.

[2] In a recent unpublished paper by Cleveland, de Keijer and Polukarov, voters consider the *relative* distance to both candidates as in our model, as well as participation costs (private communication). However, they assume voters follow Prospect Theory, and study very different problems—in particular, the computational complexity of optimizing candidates' positions .

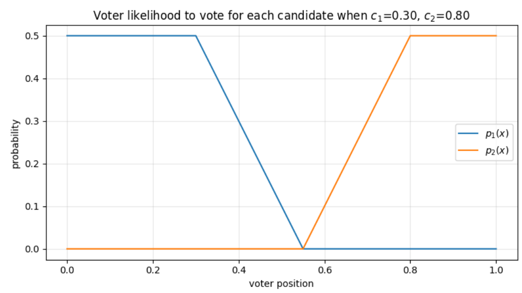

Figure 1: Vote shares by voter location under $(c_1, c_2) = (0.3, 0.8)$: for each $x \in [0, 1]$, the figure reports the fraction of voters at x who vote for candidate 1 or vote for candidate 2.

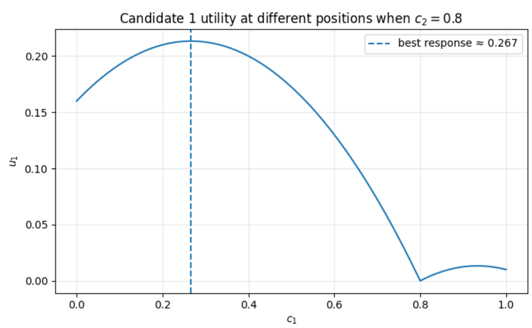

Figure 2: Candidate 1's utility $u_1(c_1, c_2)$ as a function of c_1 holding $c_2 = 0.8$ fixed. The best response occurs at approximately $c_1 \simeq 0.267$.

Example. As an illustration of the model, we consider a simple case in which both voter ideal points and voting costs are uniformly distributed on $[0, 1]$, i.e. $V, K \sim U(0, 1)$.

Figure 1 depicts voter behavior for the profile $(c_1, c_2) = (0.3, 0.8)$, with $u_1(c_1, c_2) = 0.2125$, $u_2(c_1, c_2) = 0.1625$. Fixing $c_2 = 0.8$, Figure 2 plots candidate 1's utility as a function of c_1 and indicates a best response at approximately $c_1 \simeq 0.267$. Figure 3 presents a heatmap of $u_1(c_1, c_2)$ across the full strategy space, with the candidates' best-response correspondences overlaid. The best responses intersect at two symmetric equilibria, $(\frac{1}{4}, \frac{3}{4})$ and $(\frac{3}{4}, \frac{1}{4})$, which we later show formally in Theorems 2 and 4. Intuitively, when candidates are too close to each other, many voters abstain, thus incentivizing candidates to move away from each other.

The reader can try different distributions using our online simulator: https://tinyurl.com/5ch3mwsy.

3 EQUILIBRIUM CHARACTERIZATION

3.1 Uniform Costs

First, we assume that costs are uniformly distributed, while voters' positions may not be.

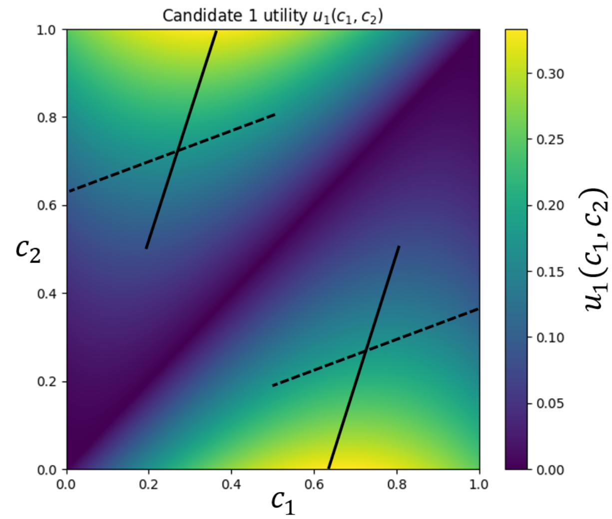

Figure 3: Heatmap of candidate 1's utility over policy profiles $(c_1, c_2) \in [0, 1]^2$. The solid line shows the best response of candidate 1 to every position of candidate 2. By super-imposing the best responses of candidate 2 (dashed lines), we can identify the equilibria at their intersections: $(\frac{1}{4}, \frac{3}{4})$ and $(\frac{3}{4}, \frac{1}{4})$.

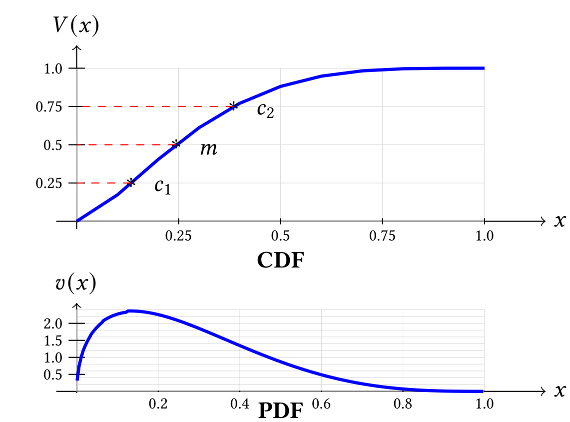

Figure 4: The (unique) symmetric equilibrium for $v(x) =$ *Beta*$(1.5, 4)$, under uniform cost distribution.

Theorem 2. *If $K \sim U(0, 1)$ and v is continuous, then a profile $(c_1, c_2) \in [0, 1]^2$ is a Nash equilibrium if and only if*

$$V\big(\min\{c_1, c_2\}\big) = V\big(\max\{c_1, c_2\}\big) - \frac{1}{2} = \frac{1}{2} V\left(\frac{c_1 + c_2}{2}\right).$$

Proof. Assume without loss of generality that $c_1 \leq c_2$ and define the midpoint $m := \frac{c_1+c_2}{2}$. Voters on the interval $[0, m]$ either vote for candidate 1 or abstain, and voters on the interval $[m, 1]$ either vote for candidate 2 or abstain. Under uniform voting costs, the fraction of voters at location x which is closer to candidate i that vote for i is

$$K\big(d_{-i}(x) - d_i(x)\big) = d_{-i}(x) - d_i(x),$$

since $\forall t \in [0, 1] : K(t) = t$.

Candidate 1. Candidate 1's utility equals

$$u_1(c_1, c_2) = \int_0^m \big(d_2(x) - d_1(x)\big)\, v(x)\, dx.$$

Evaluating the integrand piecewise, for $x \in [0, c_1]$ we have $d_1(x) = c_1 - x$ and $d_2(x) = c_2 - x$, so $d_2(x) - d_1(x) = c_2 - c_1$. For $x \in [c_1, m]$ we have $d_1(x) = x - c_1$ and $d_2(x) = c_2 - x$, so $d_2(x) - d_1(x) = c_1 + c_2 - 2x$. Therefore

$$\begin{aligned} &u_1(c_1, c_2) = \\ &= \int_0^{c_1} (c_2 - c_1)\, v(x)\, dx \;+\; \int_{c_1}^m (c_1 + c_2 - 2x)\, v(x)\, dx \\ &= (c_2 - c_1)\, V(c_1) \;+\; \int_{c_1}^m (c_1 + c_2 - 2x)\, v(x)\, dx. \end{aligned} \tag{1}$$

Since v is continuous, the function $u_1(c_1, c_2)$ is differentiable on $[0, c_2]$. Differentiating (1) with respect to c_1 and applying Leibniz' rule yields

$$\begin{aligned} &\frac{\partial u_1}{\partial c_1}(c_1, c_2) = \\ &= \frac{\partial}{\partial c_1}\Big((c_2 - c_1)V(c_1)\Big) + \frac{\partial}{\partial c_1}\left(\int_{c_1}^m (c_1 + c_2 - 2x)\, v(x)\, dx\right) \\ &= \big(-(V(c_1)) + (c_2 - c_1)v(c_1)\big) + \\ &+ \left(-(c_2 - c_1)v(c_1) + \int_{c_1}^m v(x)\, dx\right) \\ &= -V(c_1) + \big(V(m) - V(c_1)\big) = V(m) - 2V(c_1). \end{aligned}$$

Thus, any best response of candidate 1 to c_2 must satisfy

$$V(m) = 2V(c_1). \tag{2}$$

Candidate 2. Candidate 2's utility equals

$$u_2(c_1, c_2) = \int_m^1 \big(d_1(x) - d_2(x)\big)\, v(x)\, dx.$$

For $x \in [m, c_2]$, $d_1(x) = x - c_1$ and $d_2(x) = c_2 - x$, so $d_1(x) - d_2(x) = 2x - c_1 - c_2$. For $x \in [c_2, 1]$, $d_1(x) = x - c_1$ and $d_2(x) = x - c_2$, so $d_1(x) - d_2(x) = c_2 - c_1$. Therefore

$$\begin{aligned} u_2(c_1, c_2) &= \int_m^{c_2} (2x - c_1 - c_2)\, v(x)\, dx + \int_{c_2}^1 (c_2 - c_1)\, v(x)\, dx \\ &= \int_m^{c_2} (2x - c_1 - c_2)\, v(x)\, dx \;+\; (c_2 - c_1)\big(1 - V(c_2)\big). \end{aligned} \tag{3}$$

Differentiating Eq. (3) with respect to c_2 (again using Leibniz' rule) gives

$$\begin{aligned} &\frac{\partial u_2}{\partial c_2}(c_1, c_2) = \\ &= \left((c_2 - c_1)v(c_2) - \int_m^{c_2} v(x)\, dx\right) + \Big((1 - V(c_2)) - (c_2 - c_1)v(c_2)\Big) \\ &= (1 - V(c_2)) - \big(V(c_2) - V(m)\big) = 1 - 2V(c_2) + V(m). \end{aligned}$$

Hence any best response of candidate 2 to c_1 must satisfy

$$V(m) = 2V(c_2) - 1. \tag{4}$$

Equilibrium Conditions. If (c_1, c_2) is a Nash equilibrium, then both candidates best respond to each other, so Eqs. (2) and (4) hold simultaneously. Combining them yields

$$2V(c_1) = V(m) = 2V(c_2) - 1,$$

which is equivalent to

$$V(c_1) = V(c_2) - \frac{1}{2} = \frac{1}{2}\, V(m) = \frac{1}{2}\, V\!\left(\frac{c_1 + c_2}{2}\right).$$

As required. □

Existence of equilibrium. An equilibrium always exists by an application of the Intermediate Value Theorem. Let V be a continuous cumulative distribution function on $[0, 1]$. For any $c_1 \in \big[0, V^{-1}(\frac{1}{2})\big]$, define

$$c_2(c_1) \;:=\; V^{-1}\!\Big(V(c_1) + \tfrac{1}{2}\Big),$$

which is well-defined since $V(c_1) \in [0, \frac{1}{2}]$ on this domain. Next, define

$$f(c_1) \;:=\; V\!\left(\frac{c_1 + c_2(c_1)}{2}\right) - 2V(c_1).$$

By construction, f is continuous on $\big[0, V^{-1}(\frac{1}{2})\big]$. Moreover,

$$f(0) = V\!\left(\frac{V^{-1}(\frac{1}{2})}{2}\right) \geq 0,$$

and, letting $m := V^{-1}(\frac{1}{2})$ so that $c_2(m) = V^{-1}(1) = 1$,

$$f(m) = V\!\left(\frac{m+1}{2}\right) - 1 \leq 0.$$

Hence, there exists $c_1^* \in \big[0, V^{-1}(\frac{1}{2})\big]$ such that $f(c_1^*) = 0$. Setting $c_2^* = c_2(c_1^*)$ yields

$$V(c_2^*) = V(c_1^*) + \tfrac{1}{2} \qquad \text{and} \qquad V\!\left(\frac{c_1^* + c_2^*}{2}\right) = 2V(c_1^*),$$

so that (c_1^*, c_2^*) satisfies the equilibrium conditions.

Uniqueness of equilibrium. Equilibrium may not be unique, even with a symmetric voter distribution. For example, the distribution

$$v(x) = \begin{cases} \frac{5}{7}, & 0 \leq x \leq \frac{19}{50}, \\ \frac{250x - 90}{7}, & \frac{19}{50} \leq x \leq \frac{21}{50}, \\ \frac{15}{7}, & \frac{21}{50} \leq x \leq \frac{29}{50}, \\ \frac{160 - 250x}{7}, & \frac{29}{50} \leq x \leq \frac{31}{50}, \\ \frac{5}{7}, & \frac{31}{50} \leq x \leq 1. \end{cases}$$

Which is seen in Figure 5, admits three equilibria

$$(c_1, c_2) \in \left\{ \left(\frac{3}{16}, \frac{9}{16}\right),\; \left(\frac{7}{20}, \frac{13}{20}\right),\; \left(\frac{7}{16}, \frac{13}{16}\right) \right\}.$$

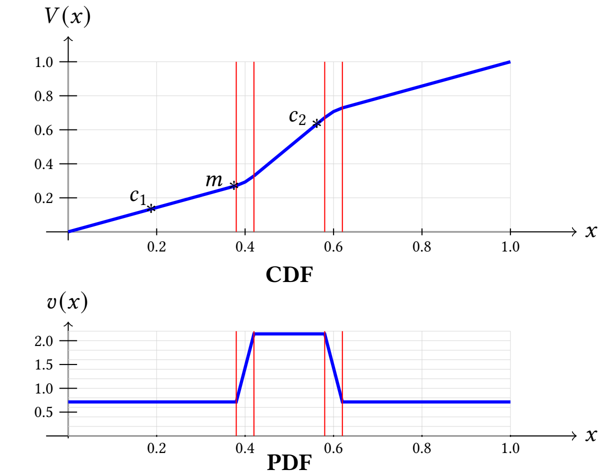

Figure 5: An example of a symmetric position distribution v with multiple equilibria. The figure shows an asymmetric equilibrium at $(3/16, 9/16)$.

Equilibria around the median. The following result follows from Theorem 2

Corollary 3. *If $K \sim U(0,1)$ and v is continuous, then any profile $(c_1, c_2) \in [0,1]^2$ satisfying*

$$V(c_1) = \frac{1}{4}, \qquad V(c_2) = \frac{3}{4}, \qquad V\left(\frac{c_1+c_2}{2}\right) = \frac{1}{2}$$

is a Nash equilibrium (see Fig. 4).

In particular, if the voter distribution is symmetric around $\frac{1}{2}$, i.e. $v(x) = v(1-x)$ for all $x \in [0,1]$, then any equilibrium satisfying the conditions of Theorem 3 must have

$$c_2 = 1 - c_1,$$

and consequently $\frac{c_1+c_2}{2} = \frac{1}{2}$, making the equilibrium symmetric and centered around $\frac{1}{2}$.

3.2 Uniform Votes

We now assume that votes are uniformly distributed, while costs may not be.

Theorem 4. *If $V \sim U(0,1)$ and k is a continuous function, the profile $(c_1, c_2) \in [0,1]^2$ is a Nash equilibrium if and only if $c_1 = 1 - c_2$ and for $\Delta = |c_2 - c_1|$*

$$K(\Delta) = (1-\Delta)k(\Delta)$$

Note that if we find Δ such that $K(\Delta) = (1-\Delta)k(\Delta)$ then the resulting equilibrium is $(\frac{1-\Delta}{2}, \frac{1+\Delta}{2})$ Proof deferred to the appendix.

Existence of equilibrium. Existence follows directly from the Intermediate Value Theorem. Define

$$f(\Delta) \;:=\; K(\Delta) - (1-\Delta)k(\Delta).$$

We have $f(0) = K(0) - k(0) = -k(0) \leq 0$ and $f(1) = K(1) = 1 > 0$. Since f is continuous on $[0,1]$, there exists $\Delta^* \in (0,1)$ such that $f(\Delta^*) = 0$, i.e., $K(\Delta^*) = (1-\Delta^*)k(\Delta^*)$. Therefore, an equilibrium always exists in this setting.

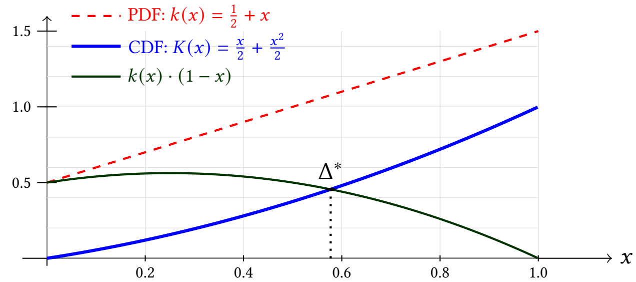

Figure 6: Visualization of the unique (symmetric) equilibrium for uniform v and $k(x) = x + \frac{1}{2}$.

Uniqueness of equilibrium. Equilibrium need not be unique. For example, if $k(\Delta) = 8\Delta^2 - \frac{16}{3}\Delta + 1$, then the equation $K(\Delta) = (1-\Delta)k(\Delta)$ admits three solutions: $\Delta \in \left\{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right\}$.

A simple sufficient condition for uniqueness is that k be non-increasing on $[0,1]$. In that case, K is non-decreasing while $(1-\Delta)k(\Delta)$ is decreasing, so the equality

$$K(\Delta) = (1-\Delta)k(\Delta)$$

can hold at most once. This condition is not necessary for uniqueness, however: for example, the increasing function

$$k(\Delta) = \Delta + \frac{1}{2}$$

still yields a unique equilibrium (see Fig. 6 and Theorem 6).

3.3 General distributions

Finally, we make no specific assumptions as to the distribution of voters or voting costs

Theorem 5. *if v, k are continuous functions than the profile $(c_1, c_2) \in [0,1]^2$ such that $c_1 \leq c_2$ is a Nash equilibrium if and only if*

1. $k(c_2 - c_1)\, V(c_1) = \displaystyle\int_{c_1}^{\frac{c_1+c_2}{2}} k(c_1 + c_2 - 2x)\, v(x)\, dx,$
2. $k(c_2 - c_1)\, \left(1 - V(c_2)\right) = \displaystyle\int_{\frac{c_1+c_2}{2}}^{c_2} k(2x - c_1 - c_2)\, v(x)\, dx.$

Proof deferred to the appendix to allow continuous reading.

We do not have full characterizations of existence or uniqueness of equilibria in the general case.

4 POLARIZATION AND VOTER TURNOUT

We next study how the distribution of voting costs shapes equilibrium polarization. We first show that (in contrast to the original Hotelling-Downs model), in our model polarized societies result in polarized candidates. The second, more interesting type of results show that even for a fixed population, increasing voting costs (i.e. reducing turnout), pushes candidates towards the extremes.

4.1 Mass polarization drives Elite Polarization

We first examine the role of voter polarization under a uniform distribution of voting costs. Recall Theorem 3, which establishes the existence of a symmetric equilibrium where

$$V(c_1) = \frac{1}{4}, \qquad \text{and} \qquad V(c_2) = \frac{3}{4}.$$

Consequently, electorates that are more polarized—in the sense of placing greater probability mass near the extremes of $[0, 1]$—admit equilibria featuring greater candidate polarization, as reflected in a larger separation between the candidates' policy positions.

While this result is intuitive, it contrasts sharply with Theorem 1 and the median voter theorem, which predict convergence toward the median policy position. The introduction of voting costs instead generates an endogenous force toward policy divergence that is absent from the standard Hotelling–Downs framework.

4.2 Elite Polarization without Mass Polarization

To isolate the effect of turnout costs, we impose a uniform voter distribution on $[0, 1]$. In particular, the median voter is located at $\frac{1}{2}$. In this setting, Theorem 4 provides a simple characterization in terms of the equilibrium platform distance $\Delta = |c_2 - c_1|$. Accordingly, we measure the degree of polarization in an equilibrium by the candidates' distance from the median voter, which under symmetry equals $\Delta/2$.

Linear cost distributions. To obtain explicit comparative statics, we consider a linear family of cost densities, $k(x) = ax + b$. The normalization condition $K(1) = \int_0^1 k(t)\,dt = 1$ implies $b = 1 - \frac{a}{2}$, and the requirement $k(x) \geq 0$ for all $x \in [0, 1]$ restricts a to the interval $(-2, 2)$.

THEOREM 6. *Assume $V \sim U(0, 1)$ and let the voting-cost density be linear,*

$$k(x) = ax + b,$$

Then a profile $(c_1, c_2) \in [0, 1]^2$ is a Nash equilibrium if and only if

$$\{c_1, c_2\} = \left\{\frac{1 - \Delta^*}{2}, \frac{1 + \Delta^*}{2}\right\},$$

where

$$\Delta^* = \begin{cases} \frac{1}{2}, & a = 0, \\ \frac{2(a-1)+\sqrt{(a-1)^2+3}}{3a}, & a \neq 0, \end{cases}$$

and when $a = 2$ there is an additional equilibrium with $\Delta^* = 0$ (i.e. $c_1 = c_2 = \frac{1}{2}$).

PROOF. Since $V \sim U(0, 1)$, we have $V(x) = x$ and, by Theorem 4, a profile (c_1, c_2) is a Nash equilibrium if and only if for $\Delta := |c_2 - c_1|$,

$$K(\Delta) = (1 - \Delta)k(\Delta). \tag{5}$$

Explicitly substituting k, K yields

$$\frac{a}{2}\Delta^2 + \left(1 - \frac{a}{2}\right)\Delta = (1 - \Delta)\left(a\Delta + 1 - \frac{a}{2}\right).$$

Expanding and rearranging gives the quadratic equation

$$3a\Delta^2 + 4(1 - a)\Delta + (a - 2) = 0. \tag{6}$$

If $a = 0$, then $k(x) \equiv 1$ and $K(\Delta) = \Delta$, so Eq. (5) becomes $\Delta = 1 - \Delta$, implying $\Delta^* = \frac{1}{2}$.

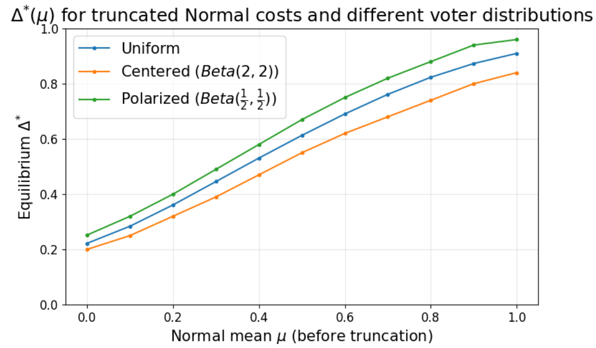

Figure 7: Polarization (measured by Δ^*) as a function of the mean μ for truncated normal voting-cost distributions $N(\mu, 0.12^2)$ on $[0, 1]$, under different spatial distributions.

Assume now $a \neq 0$. Solving Eq. (6) gives

$$\Delta = \frac{-4(1 - a) \pm \sqrt{16(1 - a)^2 - 12a(a - 2)}}{6a} = \frac{2(a - 1) \pm \sqrt{(a - 1)^2 + 3}}{3a}.$$

Among these two roots, the one that lies in $[0, 1]$ for admissible a is

$$\Delta^* = \frac{2(a - 1) + \sqrt{(a - 1)^2 + 3}}{3a},$$

with the special case $a = 0$ handled above.

Finally, since V is uniform, the equilibrium must satisfy $c_1 = 1 - c_2$, and hence any equilibrium profile with separation Δ^* is of the form

$$\{c_1, c_2\} = \left\{\frac{1 - \Delta^*}{2}, \frac{1 + \Delta^*}{2}\right\}.$$

This completes the characterization. □

Note that the equilibrium separation $\frac{\Delta^*}{2}$ is increasing in a. Consequently, larger values of a lead to equilibria with greater candidate divergence, i.e. higher polarization. Since the expected voting cost satisfies $\mathbb{E}[\kappa] = \frac{1}{2} + \frac{a}{12}$ and is therefore also increasing in a, it follows within this linear family that shifts toward higher voting costs are associated with more polarized candidate equilibria.

Normal cost distributions. In addition to the closed-form analysis, we study a distinct class of environments through simulation, focusing on truncated normal voting-cost distributions supported on $[0, 1]$. For each such distribution, we compute the equilibrium threshold Δ^* under a uniform voter distribution using a bisection routine, in accordance with the equilibrium condition in Theorem 4. We further approximate equilibria under non-uniform voter distributions via a numerical solver.

Figure 7 shows how polarization increases with the mean voting cost. As expected, more polarized societies (Beta$(\frac{1}{2}, \frac{1}{2})$) lead to more polarized outcomes, but the positive relation of costs with candidate polarization persists even under more centrally concentrated voter distributions such as Beta$(2, 2)$.

What came first? Our results show clearly that lower voter engagement (i.e., higher voting costs) can lead to elite polarization. But is the converse also true, as some political scientists claim? To test this in our model, consider a simple scenario with a uniform voter distribution and a uniform cost distribution. In this setting, the unique equilibrium is at $\left(\frac{1}{4}, \frac{3}{4}\right)$, and overall turnout is $\frac{3}{8}$. Now suppose the candidates remain *fixed*, but voting costs increase (say, according to the linear cost model with $a \in [-2, 2]$). Turnout then changes, declining from $\frac{14}{24} \cong 0.29$ at $a = -2$, to $\frac{9}{24}$ (under uniform costs), and further to $\frac{4}{24} \cong 0.08$ at $a = 2$. We can think of this change as a baseline effect, driven solely by rising voting costs and decreasing engagement.

Next, instead of keeping candidates fixed, consider turnout *in equilibrium* as we vary a over $[-2, 2]$. Although turnout still decreases, the decline is more *modest* (from 0.27 to 0.12), even as candidates become more polarized. Thus, at least in this simple scenario, abstention increases polarization, but polarization has *the opposite* effect on abstention, since the benefit from voting rises. We plan to study whether this pattern persists under other distributions as well.

5 DISCUSSION AND FURTHER RESEARCH

We provide partial characterizations of equilibrium existence and uniqueness. In particular, we establish that an equilibrium exists when either voter ideal points or voting costs are uniformly distributed. However, equilibrium in both cases may not be unique. Beyond these benchmark settings, we do not yet have general results, although we conjecture that an equilibrium always exists. A sharper characterization would help clarify how distributional features of voters and costs shape the set of candidate strategies and the comparative statics of equilibrium outcomes.

Our analysis also highlights mechanisms that generate equilibrium polarization. In contrast to the standard Hotelling–Downs benchmark, even under the simplest voting-cost specifications equilibria need not coincide with the median voter. Moreover, we find that electorates with more polarized voter distributions can support more divergent candidate platforms, and that higher voting costs may further amplify candidate separation. Since political polarization in democratic societies has increased in recent decades [6, 30] alongside declining turnout, this mechanism offers a complementary explanation for polarization, in addition to other explanations such as the roles of mainstream media [29] and social networks [21].

Beyond particular distributional families, however, we do not yet have a general comparative-statics result establishing when higher voting costs increase polarization. Identifying tractable sufficient conditions—for example in terms of stochastic dominance or shifts in the mean of the cost distribution—is therefore a promising avenue for future research.

Finally, the costly-turnout framework introduced here can be naturally combined with additional dynamics. One extension is to allow for non-linear or probabilistic voter utilities, as in [8], which may better capture how voters evaluate platforms. Another is to embed the model in a dynamic environment with repeated elections, building on iterative voting models [25]. Such a setting could generate endogenous status quo effects that shape participation incentives, in the spirit of [33].

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A SUPPLEMENTARY PROOFS

We show here the full proof for Theorem 4 and Theorem 5.

A.1 Theorem 4

Theorem 4. *If $V \sim U(0,1)$ and k is a continuous function, the profile $(c_1, c_2) \in [0,1]^2$ is a Nash equilibrium if and only if $c_1 = 1 - c_2$ and for $\Delta = |c_2 - c_1|$*

$$K(\Delta) = (1-\Delta)k(\Delta)$$

Note that if we find Δ such that $K(\Delta) = (1-\Delta)k(\Delta)$ then the resulting equilibrium is $(\frac{1-\Delta}{2}, \frac{1+\Delta}{2})$

Proof. Assume without loss of generality that $c_1 < c_2$. Let

$$\Delta := c_2 - c_1 \qquad \text{and} \qquad m := \frac{c_1 + c_2}{2}.$$

Since voting costs are not uniform, the proportion of votes that the closer candidate i receives from position x is $K\big(d_{-i}(x) - d_i(x)\big)$.

Candidate 1. Candidate 1's utility equals

$$u_1(c_1, c_2) = \int_0^m K\big(d_2(x) - d_1(x)\big)\,dx.$$

For $x \in [0, c_1]$, we have $d_1(x) = c_1 - x$ and $d_2(x) = c_2 - x$, hence $d_2(x) - d_1(x) = \Delta$. For $x \in [c_1, m]$, we have $d_1(x) = x - c_1$ and $d_2(x) = c_2 - x$, hence $d_2(x) - d_1(x) = c_1 + c_2 - 2x$. Therefore,

$$\begin{aligned} u_1(c_1, c_2) &= \int_0^{c_1} K(\Delta)\,dx \;+\; \int_{c_1}^m K(c_1 + c_2 - 2x)\,dx \\ &= c_1 K(\Delta) \;+\; \int_{c_1}^m K(c_1 + c_2 - 2x)\,dx. \end{aligned}$$

In the second integral, substitute $y = c_1 + c_2 - 2x$, so $dx = -\frac{1}{2}\,dy$. When $x = c_1$ we have $y = \Delta$, and when $x = m$ we have $y = 0$, therefore

$$\int_{c_1}^m K(c_1 + c_2 - 2x)\,dx = \frac{1}{2}\int_0^\Delta K(y)\,dy.$$

Thus

$$u_1(c_1, c_2) = c_1 K(\Delta) + \frac{1}{2}\int_0^\Delta K(y)\,dy.$$

Note that $\frac{\partial \Delta}{\partial c_1} = -1$. Differentiating gives

$$\begin{aligned} \frac{\partial u_1}{\partial c_1}(c_1, c_2) &= K(\Delta) - c_1 k(\Delta) - \frac{1}{2}K(\Delta) \\ &= \frac{1}{2}K(\Delta) - c_1 k(\Delta). \end{aligned}$$

A best response of candidate 1 therefore satisfies

$$\frac{1}{2}K(\Delta) = c_1 k(\Delta). \tag{7}$$

Candidate 2. Candidate 2's utility equals

$$u_2(c_1, c_2) = \int_m^1 K\big(d_1(x) - d_2(x)\big)\,dx.$$

For $x \in [m, c_2]$ we have $d_1(x) = x - c_1$ and $d_2(x) = c_2 - x$, hence $d_1(x) - d_2(x) = 2x - c_1 - c_2$. For $x \in [c_2, 1]$ we have $d_1(x) = x - c_1$ and $d_2(x) = x - c_2$, hence $d_1(x) - d_2(x) = \Delta$. Therefore,

$$\begin{aligned} u_2(c_1, c_2) &= \int_m^{c_2} K(2x - c_1 - c_2)\,dx \;+\; \int_{c_2}^1 K(\Delta)\,dx \\ &= \int_m^{c_2} K(2x - c_1 - c_2)\,dx \;+\; (1 - c_2)K(\Delta). \end{aligned}$$

Substituting $y = 2x - c_1 - c_2$ in the first term (so $dx = \frac{1}{2}\,dy$, with $y = 0$ at $x = m$ and $y = \Delta$ at $x = c_2$) yields

$$\int_m^{c_2} K(2x - c_1 - c_2)\,dx = \frac{1}{2}\int_0^\Delta K(y)\,dy,$$

and thus

$$u_2(c_1, c_2) = (1 - c_2)K(\Delta) + \frac{1}{2}\int_0^{\Delta} K(y)\,dy.$$

Note that $\frac{\partial \Delta}{\partial c_2} = 1$. Differentiating gives

$$\frac{\partial u_2}{\partial c_2}(c_1, c_2) = -K(\Delta) + (1 - c_2)k(\Delta) + \frac{1}{2}K(\Delta)$$
$$= (1 - c_2)k(\Delta) - \frac{1}{2}K(\Delta).$$

A best response of candidate 2 therefore satisfies

$$\frac{1}{2}K(\Delta) = (1 - c_2)k(\Delta). \tag{8}$$

Equilibrium conditions. At a Nash equilibrium with $c_1 < c_2$ and best responses, conditions (7)–(8) must both hold, implying

$$c_1 k(\Delta) = (1 - c_2)k(\Delta) \quad \Rightarrow \quad c_1 = 1 - c_2,$$

and hence $c_1 + c_2 = 1$ and $c_1 = \frac{1-\Delta}{2}$, $c_2 = \frac{1+\Delta}{2}$. Substituting into Eq. (7) yields

$$\frac{1}{2}K(\Delta) = \frac{1-\Delta}{2}\,k(\Delta) \quad \Longleftrightarrow \quad K(\Delta) = (1 - \Delta)k(\Delta),$$

As required. □

A.2 Theorem 5

Theorem 5. *if v, k are continuous functions than the profile $(c_1, c_2) \in [0, 1]^2$ such that $c_1 \le c_2$ is a Nash equilibrium if and only if*

$$1. \quad k(c_2 - c_1)\,V(c_1) = \int_{c_1}^{\frac{c_1+c_2}{2}} k(c_1 + c_2 - 2x)\,v(x)\,dx,$$

$$2. \quad k(c_2 - c_1)\,\big(1 - V(c_2)\big) = \int_{\frac{c_1+c_2}{2}}^{c_2} k(2x - c_1 - c_2)\,v(x)\,dx.$$

Proof. Fix a profile $(c_1, c_2) \in [0, 1]^2$ with $c_1 \le c_2$, and let

$$\Delta := c_2 - c_1, \qquad m := \frac{c_1 + c_2}{2}.$$

Since voting costs are not uniform, the proportion of votes that the closer candidate i recieves from position x is $K\big(d_{-i}(x) - d_i(x)\big)$.

Candidate 1. Candidate 1's utility equals

$$u_1(c_1, c_2) = \int_0^m K\big(d_2(x) - d_1(x)\big)\,v(x)\,dx.$$

For $x \in [0, c_1]$ we have $d_1(x) = c_1 - x$ and $d_2(x) = c_2 - x$, hence $d_2(x) - d_1(x) = \Delta$. For $x \in [c_1, m]$ we have $d_1(x) = x - c_1$ and $d_2(x) = c_2 - x$, hence $d_2(x) - d_1(x) = c_1 + c_2 - 2x$. Therefore,

$$u_1(c_1, c_2) = \int_0^{c_1} K(\Delta)\,v(x)\,dx \;+\; \int_{c_1}^{m} K(c_1 + c_2 - 2x)\,v(x)\,dx. \tag{9}$$

Assume v and k are continuous. Differentiating Eq. (9) with respect to c_1 and applying Leibniz' rule yields

$$\begin{aligned}
&\frac{\partial u_1}{\partial c_1}(c_1, c_2) = \\
&K(\Delta)v(c_1) \;-\; k(\Delta)\int_0^{c_1} v(x)\,dx \\
&+\; \big[-K(c_1 + c_2 - 2x)v(x)\big]_{x=c_1} \\
&+\int_{c_1}^{m} k(c_1 + c_2 - 2x)\,v(x)\,dx \;+\; K(0)v(m)\cdot\frac{\partial m}{\partial c_1}.
\end{aligned}$$

Since the cost support is $[0, 1]$, we have $K(0) = 0$, so the last term vanishes. Moreover, $\int_0^{c_1} v(x)\,dx = V(c_1)$ and the lower-limit term equals $-K(\Delta)v(c_1)$. Hence,

$$\frac{\partial u_1}{\partial c_1}(c_1, c_2) = \int_{c_1}^{m} k(c_1 + c_2 - 2x)\,v(x)\,dx \;-\; k(\Delta)\,V(c_1).$$

Thus, at the best response

$$\frac{\partial u_1}{\partial c_1}(c_1, c_2) = 0 \iff k(c_2 - c_1)\, V(c_1) = \int_{c_1}^{\frac{c_1+c_2}{2}} k(c_1 + c_2 - 2x)\, v(x)\, dx,$$

which is condition (1).

Candidate 2. Candidate 2's utility equals

$$u_2(c_1, c_2) = \int_m^1 K\big(d_1(x) - d_2(x)\big)\, v(x)\, dx.$$

For $x \in [m, c_2]$ we have $d_1(x) = x - c_1$ and $d_2(x) = c_2 - x$, hence $d_1(x) - d_2(x) = 2x - c_1 - c_2$. For $x \in [c_2, 1]$ we have $d_1(x) = x - c_1$ and $d_2(x) = x - c_2$, hence $d_1(x) - d_2(x) = \Delta$. Therefore,

$$u_2(c_1, c_2) = \int_{c_2}^1 K(\Delta)\, v(x)\, dx \ + \int_m^{c_2} K(2x - c_1 - c_2)\, v(x)\, dx. \tag{10}$$

Differentiating Eq. (10) with respect to c_2 and applying Leibniz' rule gives

$$\frac{\partial u_2}{\partial c_2}(c_1, c_2) = k(\Delta) \int_{c_2}^1 v(x)\, dx \ - \int_m^{c_2} k(2x - c_1 - c_2)\, v(x)\, dx.$$

where again the boundary terms involving $K(0)$ cancel because $K(0) = 0$. Since $\int_{c_2}^1 v(x)\, dx = 1 - V(c_2)$, the first-order condition becomes

$$\frac{\partial u_2}{\partial c_2}(c_1, c_2) = 0 \iff k(c_2 - c_1)\big(1 - V(c_2)\big) = \int_{\frac{c_1+c_2}{2}}^{c_2} k(2x - c_1 - c_2)\, v(x)\, dx.$$

which is condition (2).

Equilibrium conditions. If (c_1, c_2) is a Nash equilibrium with $c_1 \le c_2$, then both first-order conditions must hold, yielding (1) and (2). Conversely, if (1) and (2) hold, then both candidates satisfy the first-order conditions against the opponent's position. □